RISK AND RATES OF RETURN

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INVESTMENT RETURNS

- With most investments, an individual or business spends money today with the expectation of earning even more money in the future.
- The concept of *return* provides investors with a convenient way of expressing the financial performance of an investment.

- The solution to the scale and timing problems is to express investment results as *rates of return, or percentage returns*.
- *For example, the rate of return on the* 1-year stock investment, when \$1,100 is received after one year, is 10 percent:

$$\text{Rate of return} = \frac{\text{Amount received} - \text{Amount invested}}{\text{Amount invested}}$$

$$= \frac{\text{Dollar return}}{\text{Amount invested}} = \frac{\$100}{\$1{,}000}$$

$$= 0.10 = 10\%.$$

- The rate of return calculation “standardizes” the return by considering the return per unit of investment.
- In this example, the return of 0.10, or 10 percent, indicates that each dollar invested will earn 0.10(\$1.00) \$0.10..

STAND-ALONE RISK

- **Risk is defined in *Webster's as "a hazard; a peril; exposure to loss or injury."***
- Thus, risk refers to the chance that some unfavorable event will occur.
- An asset's risk can be analyzed in two ways: (1) on a stand-alone basis, wherethe asset is considered in isolation, and (2) on a portfolio basis, where the asset is held as one of a number of assets in a portfolio.

PROBABILITY DISTRIBUTIONS

- An event's *probability is defined as the chance that the event will occur. For example*, a weather forecaster might state, "There is a 40 percent chance of rain today and a 60 percent chance that it will not rain."
- **Probability Distribution -** A listing of all possible outcomes, or events, with a probability (chance of occurrence) assigned to each outcome.

Probability Distributions for Martin Products and U.S. Water

DEMAND FOR THE COMPANY'S PRODUCTS	PROBABILITY OF THIS DEMAND OCCURRING	RATE OF RETURN ON STOCK IF THIS DEMAND OCCURS	
		MARTIN PRODUCTS	U.S. WATER
Strong	0.3	100%	20%
Normal	0.4	15	15
Weak	0.3	(70)	10
	1.0		

- There is a 30 percent chance of strong demand, in which case both companies will have high earnings, pay high dividends, and enjoy capital gains.
- There is a 40 percent probability of normal demand and moderate returns, and there is a 30 percent probability of weak demand, which will mean low earnings and dividends as well as capital losses.

EXPECTED RATE OF RETURN

- If we multiply each possible outcome by its probability of occurrence and then sum these products, as in Table 2, we have a *weighted average of outcomes*.
- The weights are the probabilities, and the weighted average is the **expected rate of return, $\hat{k}$, called "k-hat."**
- The expected rates of return for both Martin Products and U.S. Water are shown in Table 6-2 to be 15 percent. This type of table is known as a *payoff matrix*.

Calculation of Expected Rates of Return: Payoff Matrix

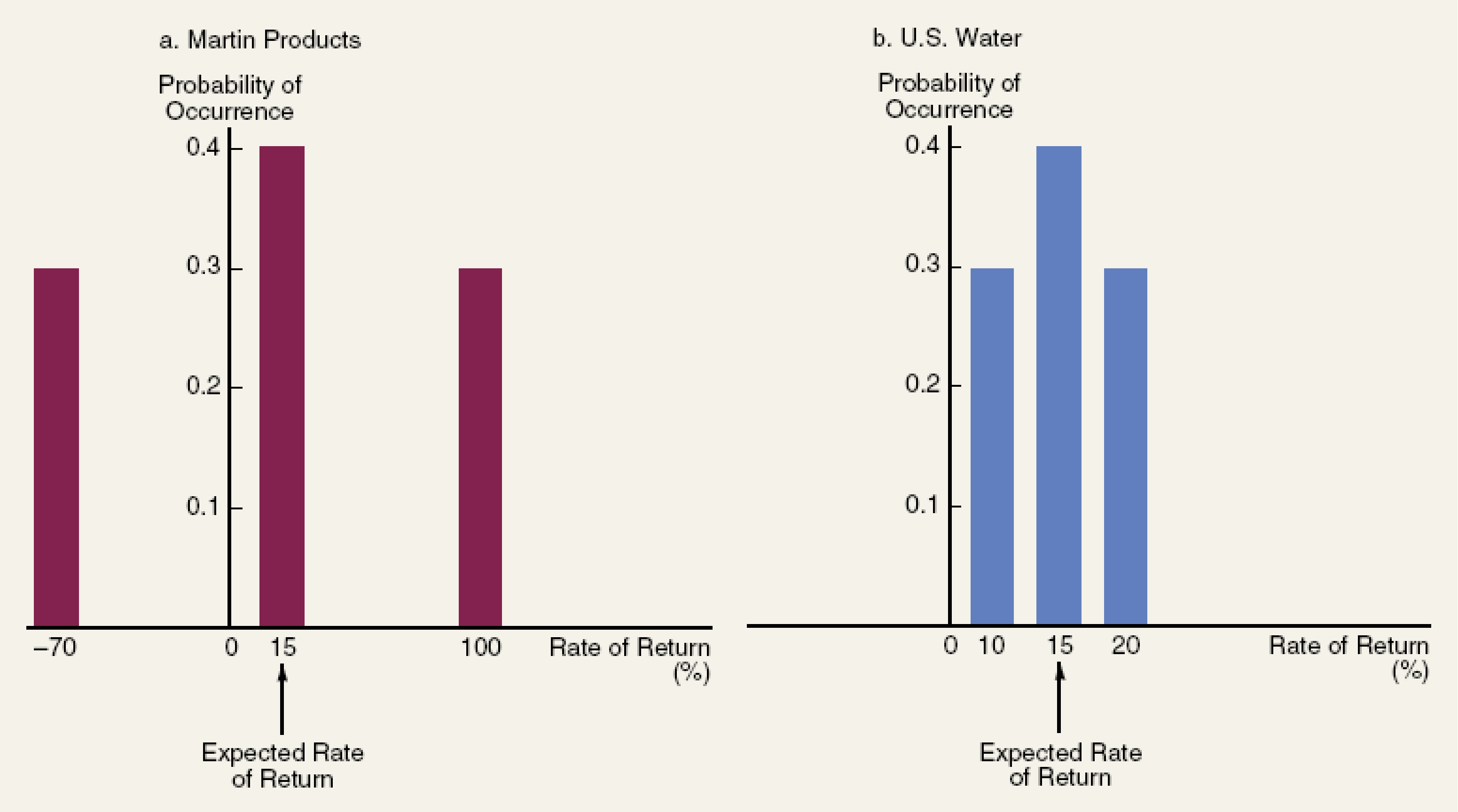
		MARTIN PRODUCTS		U.S. WATER	
DEMAND FOR THE COMPANY'S PRODUCTS (1)	PROBABILITY OF THIS DEMAND OCCURRING (2)	RATE OF RETURN IF THIS DEMAND OCCURS (3)	PRODUCT: (2) × (3) = (4)	RATE OF RETURN IF THIS DEMAND OCCURS (5)	PRODUCT: (2) × (5) = (6)
Strong	0.3	100%	30%	20%	6%
Normal	0.4	15	6	15	6
Weak	0.3	(70)	(21)	10	3
	1.0		$\hat{k}$ = 15%		$\hat{k}$ = 15%

The expected rate of return calculation can also be expressed as an equation that does the same thing as the payoff matrix table

$$\text{Expected rate of return} = \hat{k} = P_1k_1 + P_2k_2 + \cdots + P_nk_n$$

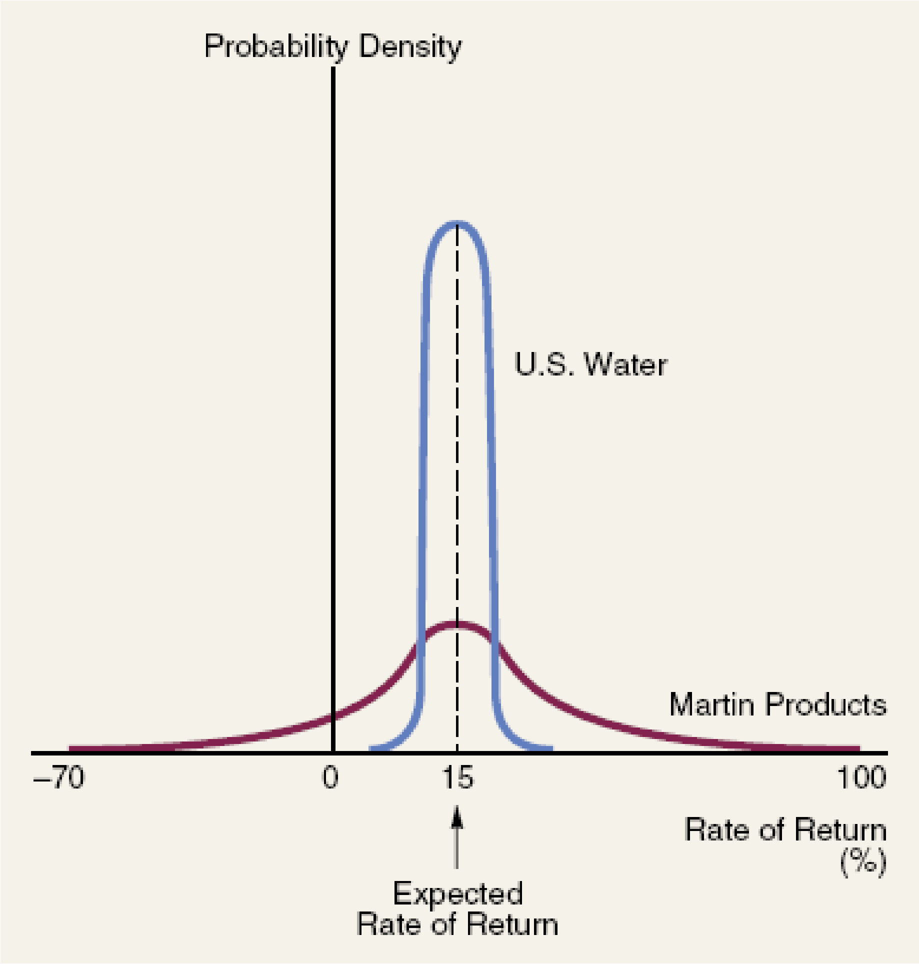
$$= \sum_{i=1}^{n} P_ik_i.$$

Probability Distributions of Martin Products' and U.S. Water's Rates of Return



MEASURING STAND-ALONE RISK: THE STANDARD DEVIATION

Continuous Probability Distributions of Martin Products' and U.S. Water's Rates of Return



THE STANDARD DEVIATION

- To be most useful, any measure of risk should have a definite value—we need a measure of the tightness of the probability distribution. One such measure is the **standard deviation, the symbol for which is , pronounced "sigma."**
- The smaller the standard deviation, the tighter the probability distribution, and, accordingly, the lower the riskiness of the stock.

Calculating Martin Products' Standard Deviation

$k_i - \hat{k}$ (1)	$(k_i - \hat{k})^2$ (2)	$(k_i - \hat{k})^2 P_i$ (3)
$100 - 15 = 85$	7,225	$(7{,}225)(0.3) = 2{,}167.5$
$15 - 15 = 0$	0	$(0)(0.4) = 0.0$
$-70 - 15 = -85$	7,225	$(7{,}225)(0.3) = \underline{2{,}167.5}$
		Variance $= \sigma^2 = \underline{\underline{4{,}335.0}}$

$$\text{Standard deviation} = \sigma = \sqrt{\sigma^2} = \sqrt{4{,}335} = 65.84\%.$$

1. Calculate the expected rate of return:

$$\text{Expected rate of return} = \hat{k} = \sum_{i=1}^{n} P_i k_i.$$

For Martin, we previously found $\hat{k} = 15\%$.

2. Subtract the expected rate of return ($\hat{k}$) from each possible outcome (k_i) to obtain a set of deviations about $\hat{k}$ as shown in Column 1 of Table 6-3:

$$\text{Deviation}_i = k_i - \hat{k}.$$

Square each deviation, then multiply the result by the probability of occurrence for its related outcome, and then sum these products to obtain the **variance** of the probability distribution as shown in Columns 2 and 3 of the table:

$$\text{Variance} = \sigma^2 = \sum_{i=1}^{n} (k_i - \hat{k})^2 P_i. \qquad \textbf{(6-2)}$$

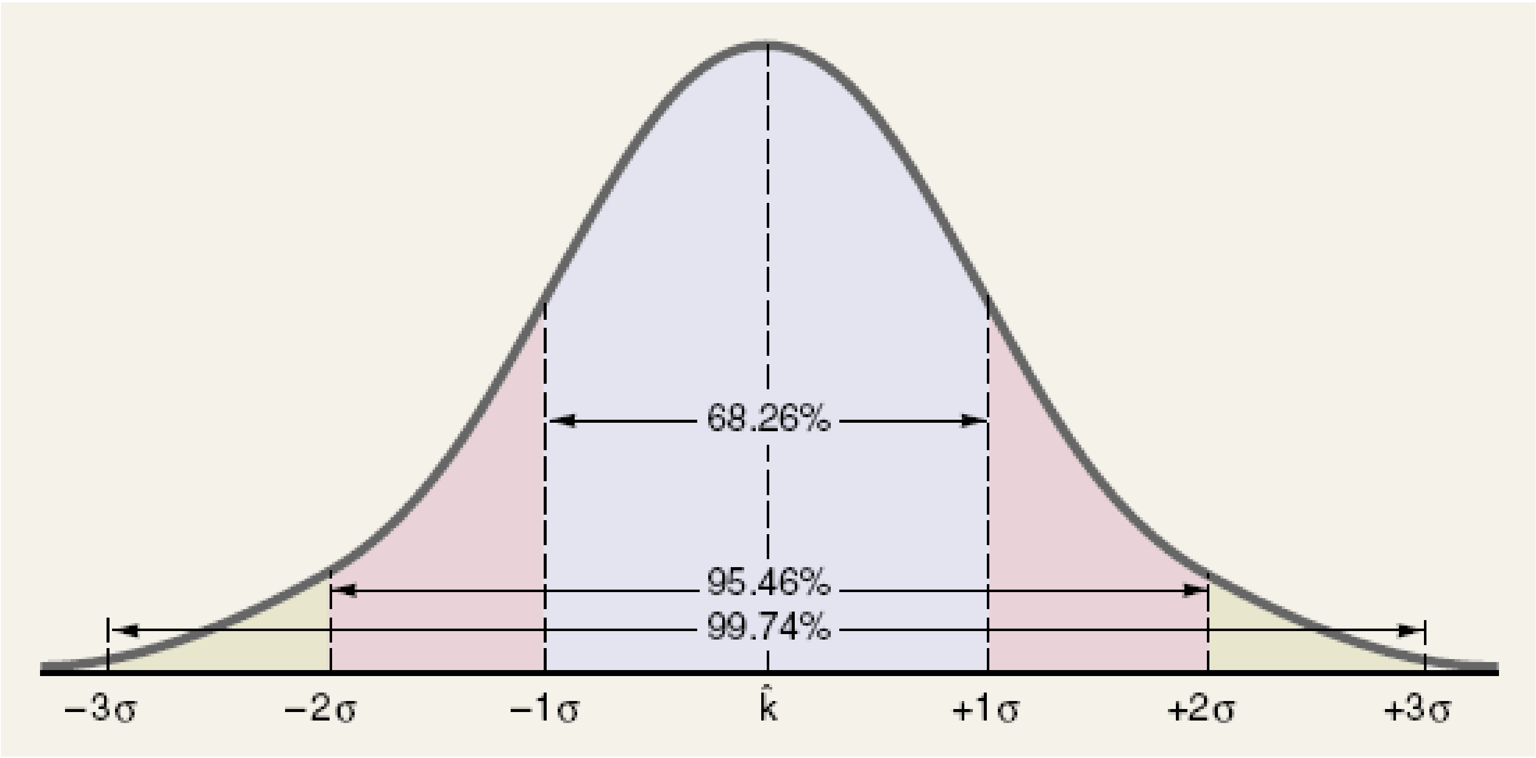
Finally, find the square root of the variance to obtain the standard deviation:

$$\text{Standard deviation} = \sigma = \sqrt{\sum_{i=1}^{n} (k_i - \hat{k})^2 P_i}. \qquad \textbf{(6-3)}$$

If a probability distribution is normal, the *actual return will be*:

- *within 1* standard deviation of the *expected return 68.26 percent of the time*.
- *within 2* standard deviation of the *expected return 95,46 percent of the time*.
- *within 3* standard deviation of the *expected return 99,74 percent of the time*.

Probability Ranges for a Normal Distribution



THE COEFFICIENT OF VARIATION

$$\text{Coefficient of variation} = \text{CV} = \frac{\sigma}{\hat{k}}.$$

- *The coefficient of variation shows the risk per unit of return, and it provides a more meaningful basis for comparison when the expected returns on two alternatives are not the same.*

RISK AVERSION AND REQUIRED RETURNS

- **Risk Aversion -** Risk-averse investors dislike risk and require higher rates of return as an inducement to buy riskier securities.
- **Risk Premium, RP -** The difference between the expected rate of return on a given risky asset and that on a less risky asset.

  In a market dominated by risk-averse investors, riskier securities must have higher expected returns, as estimated by the marginal investor, than less risky securities. If this situation does not exist, buying and selling in the market will force it to occur.

LITERATURE:

- SCHWAB, CH., LYNCH, M.: Fundamentals of financial management, Brigham&Houston, tenth editional, 2003, p.787

THANKS FOR ATTENTION