TIME VALEU OF MONEY

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CONTENS

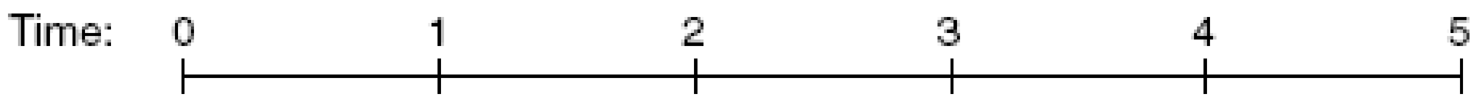
- For financial managers to have a clear understanding of the time value of money and its impact on stock prices. These concepts are discussed in this lesson, where we show how the timing of cash flows affects asset values and rates of return.

CONTENS

- TIME LINES
- FUTURE VALUE
- PRESENT VALUE
- SOLVING FOR INTEREST RATE AND TIME
- FUTURE VALUE OF AN ANNUITY
- PRESENT VALUE OF AN ANNUITY

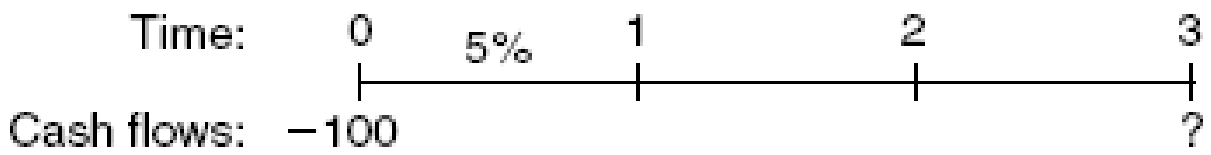
TIME LINES

- One of the most important tools in time value analysis is the **time line, which** is used by analysts to help visualize what is happening in a particular problem and then to help set up the problem for solution.
- To illustrate the time line concept, consider the following diagram:



- Time 0 is today; Time 1 is one period from today, or the end of Period 1; Time 2 is two periods from today, or the end of Period 2; and so on. Thus, the numbers above the tick marks represent end-of-period values.
- Often the periods are years, but other time intervals such as semiannual periods, quarters, months, or even days can be used.

- Cash flows are placed directly below the tick marks, and interest rates are shown directly above the time line. Unknown cash flows, which you are trying to find in the analysis, are indicated by question marks.
- Now consider the following time line:



- Here the interest rate for each of the three periods is 5 percent; a single amount (or lump sum) cash **outflow is made at Time 0; and the Time 3 value is an unknown inflow. Since the initial $100 is an outflow (an investment), it has a** minus sign. Since the Period 3 amount is an inflow, it does not have a minus sign, which implies a plus sign.

FUTURE VALUE

- A dollar in hand today is worth more than a dollar to be received in the future because, if you had it now, you could invest it, earn interest, and end up with more than one dollar in the future. The process of going from today's values, or present values (PVs), to future values (FVs) is called **compounding.**

- To illustrate, suppose you deposit $100 in a bank that pays 5 percent interest each year. How much would you have at the end of one year? To begin, we define the following terms:

- PV = present value, or beginning amount, in your account. Here PV = $100.
- i = interest rate the bank pays on the account per year. The interest earned is based on the balance at the beginning of each year, and we assume that it is paid at the end of the year. Here i = 5%, or, expressed as a decimal, i = 0.05.
- INT = dollars of interest you earn during the year Beginning amount i. Here INT = $100(0.05) = $5.
- FVn =future value, or ending amount, of your account at the end of n years. Whereas PV is the value now, or the *present value, FVn is the value n* years into the *future, after the interest earned has been added to the account*.
- n = number of periods involved in the analysis. Here n = 1.

In our example, n = 1, so FV_n can be calculated as follows:

$$
\begin{aligned}
FV_n = FV_1 &= PV + INT \\
&= PV + PV(i) \\
&= PV(1 + i) \\
&= \$100(1 + 0.05) = \$100(1.05) = \$105.
\end{aligned}
$$

Thus, the **future value (FV) at the end of one year, FV1, equals the present** value multiplied by 1 plus the interest rate, so you will have $105 after one year.

Future Value (FV) - The amount to which a cash flow or series of cash flows will grow over a given period of time when
compounded at a given interest rate.

What would you end up with if you left your $100 in the account for five years? Here is a time line set up to show the amount at the end of each year

	0	1	2	3	4	5
	5%					
Initial deposit:	−100	FV_1 = ?	FV_2 = ?	FV_3 = ?	FV_4 = ?	FV_5 = ?
Interest earned:		5.00	5.25	5.51	5.79	6.08
Amount at the end of each period = FV_n:		105.00	110.25	115.76	121.55	**127.63**

Note the following points:

(1) You start by depositing $100 in the account—this is shown as an outflow at t 0.

(2) You earn $100(0.05) = $5 of interest during the first year, so the amount at the end of Year 1 (or t 1) is $100 + $5 = $105.

(3) You start the second year with $105, earn $5.25 on the now larger amount, and end the second year with $110.25. Your interest during Year 2, $5.25, is higher than the first year’s interest, $5, because you earned $5(0.05) = $0.25 interest on the first year’s interest.

(4) This process continues, and because the beginning balance is higher in each succeeding year, the annual interest earned increases.

(5) The total interest earned, $27.63, is reflected in the final balance at t=5, $127.63.

Note that the value at the end of Year 2, \$110.25, is equal to

$$\begin{aligned} FV_2 &= FV_1(1 + i) \\ &= PV(1 + i)(1 + i) \\ &= PV(1 + i)^2 \\ &= \$100(1.05)^2 = \$110.25. \end{aligned}$$

Continuing, the balance at the end of Year 3 is

$$\begin{aligned} FV_3 &= FV_2(1 + i) \\ &= PV(1 + i)^3 \\ &= \$100(1.05)^3 = \$115.76, \end{aligned}$$

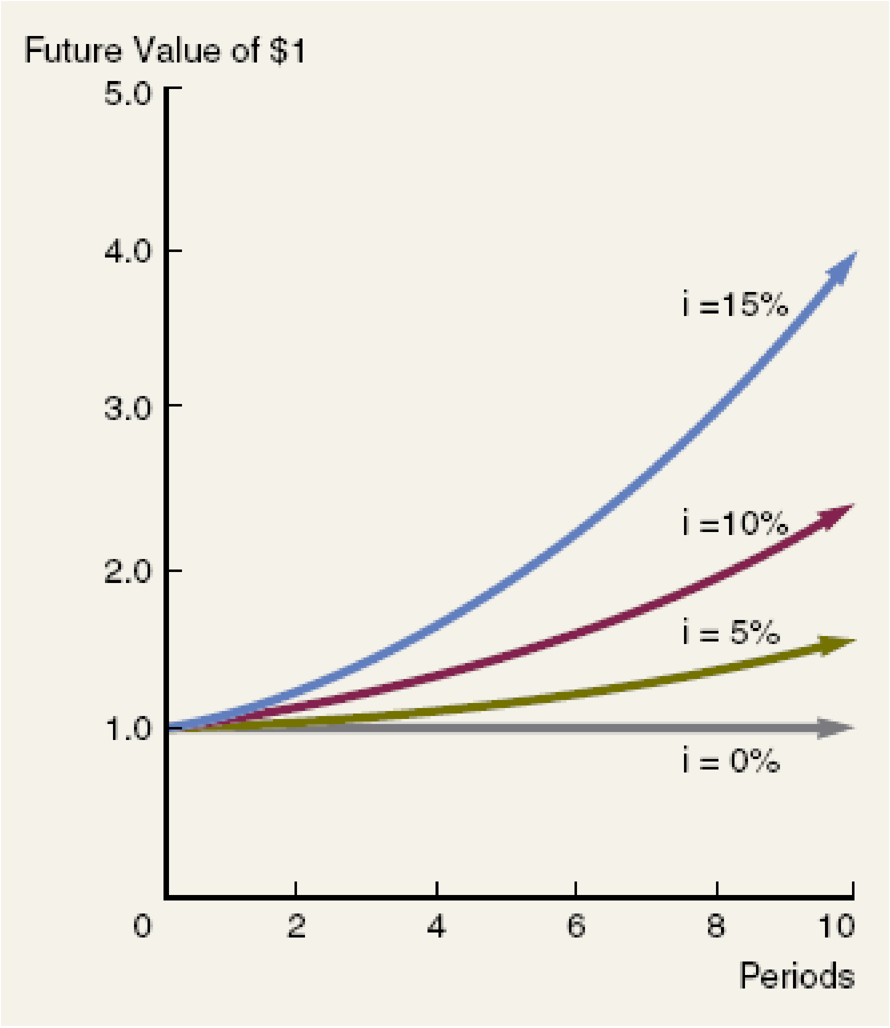
and

$$FV_5 = \$100(1.05)^5 = \$127.63.$$

In general, the future value of an initial lump sum at the end of n years can be found by applying Equation 7-1:

$$FV_n = PV(1 + i)^n. \tag{7-1}$$

Relationships among Future Value, Growth, Interest Rates, and Time



- Figure shows how $1 (or any other lump sum) grows over time at various interest rates.
- The higher the rate of interest, the faster the rate of growth. The interest rate is, in fact, a growth rate: If a sum is deposited and earns 5 percent interest, then the funds on deposit will grow at a rate of 5 percent per period.
- Note also that time value concepts can be applied to anything that is growing—sales, population, earnings per share, your future salary, or whatever.

PRESENT VALUE

- Suppose you have some extra cash, and you have a chance to buy a low-risk security that will pay $127.63 at the end of five years. Your local bank is currently offering 5 percent interest on five-year certificates of deposit (CDs), and you regard the security as being exactly as safe as a CD.
- The 5 percent rate is defined as your **opportunity cost rate, or the rate of return you could earn on an** alternative investment of similar risk. How much should you be willing to pay for the security?

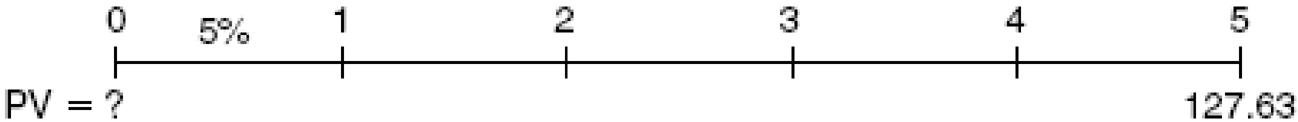
- From the future value example presented in the previous section, we saw that an initial amount of $100 invested at 5 percent per year would be worth $127.63 at the end of five years. As we will see in a moment, you should be indifferent between $100 today and $127.63 at the end of five years.
- The $100 is defined as the **present value, or PV, of $127.63 due in five years** when the opportunity cost rate is 5 percent.
- If the price of the security were less than $100, you should buy it, because its price would then be less than the $100 you would have to spend on a similar-risk alternative to end up with $127.63 after five years.

- In general, *the present value of a cash flow due n years in the future is the amount which, if it were on hand today, would grow to equal the future amount. Since $100* would grow to $127.63 in five years at a 5 percent interest rate, $100 is the present value of $127.63 due in five years when the opportunity cost rate is 5 percent.

- **Fair (Equilibrium) Value -** The price at which investors are indifferent between buying or selling a security.

Finding present values is called **discounting**, and it is simply the reverse of compounding — if you know the PV, you can compound to find the FV, while if you know the FV, you can discount to find the PV. When discounting, you would follow these steps:

Time Line:



Equation:

To develop the discounting equation, we begin with the future value equation, Equation 7-1:

$$FV_n = PV(1 + i)^n = PV(FVIF_{i,n}). \qquad \textbf{(7-1)}$$

Next, we solve it for PV in several equivalent forms:

$$PV = \frac{FV_n}{(1 + i)^n} = FV_n\left(\frac{1}{1 + i}\right)^n = FV_n(PVIF_{i,n}). \qquad \textbf{(7-2)}$$

The last form of Equation 7-2 recognizes that the interest factor $PVIF_{i,n}$ is equal to the term in parentheses in the second version of the equation.

1. Numerical Solution

0	5%	1	2	3	4	5
−100 =	←	105.00 ←	110.25 ←	115.76 ←	121.55 ←	127.63
		÷ 1.05	÷ 1.05	÷ 1.05	÷ 1.05	÷ 1.05

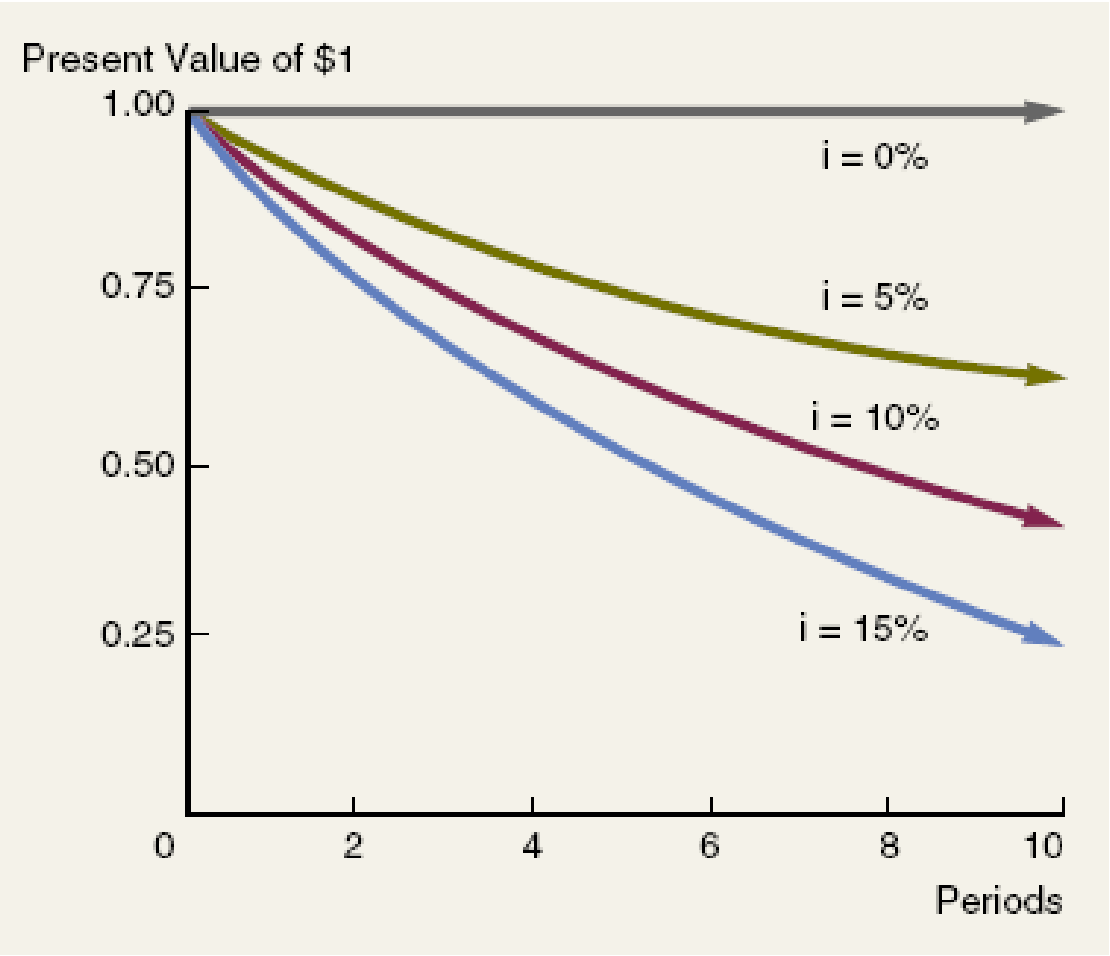
Divide \$127.63 by 1.05 five times, or by $(1.05)^5$, to find PV = \$100.

The next graph shows :

(1)that the present value of a sum to be received at some future date decreases and approaches zero as the payment date is extended further into the future, and

(2)that the rate of decrease is greater the higher the interest (discount) rate. At relatively high interest rates, funds due in the future are worth very little today, and even at a relatively low discount rate, the present value of a sum due in the very distant future is quite small.

Relationships among Present Value, Interest Rates, and Time



SOLVING FOR INTEREST RATE AND TIME

At this point, you should realize that compounding and discounting are related, and that we have been dealing with one equation that can be solved for either the FV or the PV.

FV Form:

$$FV_n = PV(1 + i)^n. \qquad (7\text{-}1)$$

PV Form:

$$PV = \frac{FV_n}{(1 + i)^n} = FV_n\left(\frac{1}{1 + i}\right)^n. \qquad (7\text{-}2)$$

- There are four variables in these equations—PV, FV, i, and n—and if you know the values of any three, you can find the value of the fourth. Thus far, we have always given you the interest rate (i) and the number of years (n), plus either the PV or the FV. In many situations, though, you will need to solve for either i or n, as we discuss below.

SOLVING FOR I

- Suppose you can buy a security at a price of \$78.35, and it will pay you \$100 after five years. Here you know PV, FV, and n, and you want to find i, the interest rate you would earn if you bought the security. Problems such as this are solved as follows:

Time Line:

Equation:

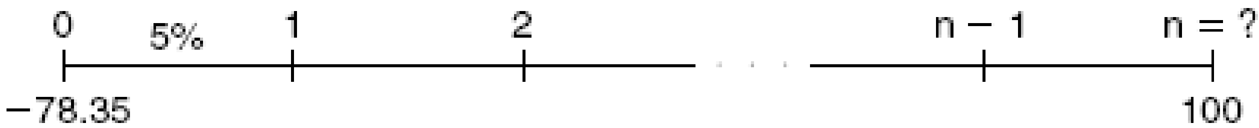
$$FV_n = PV(1 + i)^n \qquad (7\text{-}1)$$

$$\$100 = \$78.35(1 + i)^5. \text{ Solve for i.}$$

SOLVING FOR N

- Suppose you know that a security will provide a return of 5 percent per year, that it will cost $78.35, and that you will receive $100 at maturity, but you do not know when the security matures. Thus, you know PV, FV, and i, but you do not know n, the number of periods. Here is the situation:

Time Line:



Equation:

$$FV_n = PV(1 + i)^n \tag{7-1}$$

$$\$100 = \$78.35(1.05)^n. \text{ Solve for n.}$$

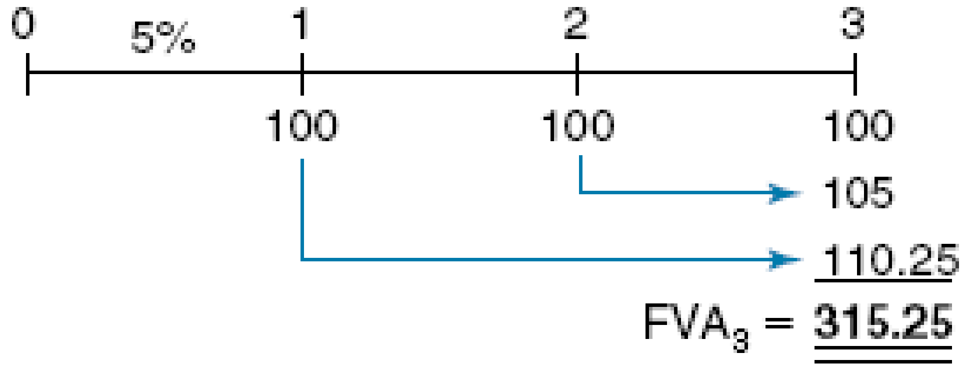
FUTURE VALUE OF AN ANNUITY

- An **annuity is a series of equal payments made at fixed intervals for a specified** number of periods. For example, $100 at the end of each of the next three years is a three-year annuity. The payments are given the symbol PMT, and they can occur at either the beginning or the end of each period. If the payments occurat the *end of each period, as they typically do, the annuity is called an* ***ordinary*** **annuity.**
- If payments are made at the *beginning of* each period, the annuity is an **annuity due**

ORDINARY ANNUITIES

- Since ordinary annuities are more common in finance, when the term "annuity" is used, you should assume that the payments occur at the end of each period unless otherwise noted.

Time Line:

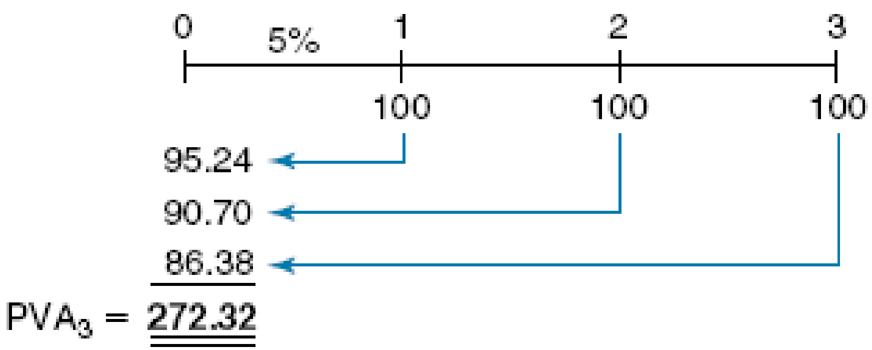


PRESENT VALUE OF AN ANNUITY

Ordinary Annuities

If the payments come at the end of each year, then the annuity is an ordinary annuity, and it would be set up as follows:

Time Line:



PVAn - The present value of an annuity of n periods.

LITERATURE:

- SCHWAB, CH., LYNCH, M.: Fundamentals of financial management, Brigham&Houston, tenth editional, 2003, p.787

THANKS FOR ATTENTION