Na prednáške:

Pr.1: Vyjadriť inverznú funkciu (k exponenciálnej → logaritmická)

f: $y = 5 + 10^{x-4}$

f^{-1}: $x = 5 + 10^{y-4}$ / -5

$x - 5 = 10^{y-4}$ / ∘ $\log_{10}$

$x - 5 = log_{10}(10)^{y-4}$

$log_{10}(x-5) = y - 4$ / +4

$4 + \log(x-5) = y$

f^{-1}: $y = 4 + \log(x-5)$ D(f^{-1}) = (5, ∞)

Pr.2: Vyjadriť inverznú funkciu (k logaritmickej → exponenciálna)

f: $y = -2 + log_3(x+6)$, podmienka: x + 6 > 0, D(f) = (-6, ∞)

f^{-1}: $x = -2 + log_3(y+6)$ /+2

$x + 2 = log_3(y+6)$ / ∘ $3^{(\,)}$ odlogaritmujeme

$3^{(x+2)} = 3^{log_3(y+6)}$

$3^{(x+2)} = y + 6$ / − 6

$-6 + 3^{(x+2)} = y$

f^{-1}: $y = -6 + 3^{(x+2)}$

Samostatné štúdium:

Pr. Zistite oblasť definície a vyjadrite inverznú funkciu $f: y = 2 - 8 \cdot \log_3(4-3x)$

Riešenie:

$4 - 3x > 0 \quad \Rightarrow \quad 4 > 3x \quad \Rightarrow \quad \frac{4}{3} > x \quad \Rightarrow \quad D(f) = (-\infty, \frac{4}{3})$

f^{-1}: $x = 2 - 8\log_3(4-3y)$

$x - 2 = -8\log_3(4-3y)$ / (-1)

$-x + 2 = 8\log_3(4-3y)$

$\frac{2-x}{8} = \log_3(4-3y)$ / ∘ $3^{(\,)}$

$$3^{\frac{2-x}{8}} = 4-3y$$

$$3y = 4-3^{\frac{2-x}{8}}$$

$$f^{-1}: \quad y = \frac{4-3^{\frac{2-x}{8}}}{3}$$

Pr. Zistite oblasť definície funkcie $f: y = 1+4\arcsin\left(\frac{3x-2}{4}\right)$

Riešenie:

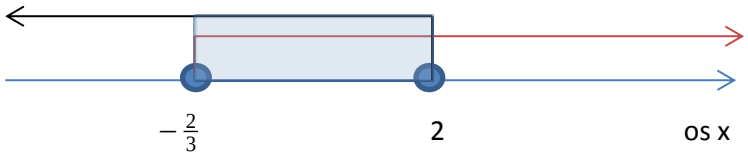
$-1 \le \frac{3x-2}{4}$ a zároveň $\frac{3x-2}{4} \le 1$

$-4 \le 3x-2$ $\quad$ $3x-2 \le 4$

$-2 \le 3x$ $\quad$ $3x \le 6$

$-\frac{2}{3} \le x$ $\quad$ $x \le 2$

$D(f) = <-\frac{2}{3}, \infty) \cap (-\infty, 2> \quad \Rightarrow \quad D(f) = \left\langle -\frac{2}{3}, 2 \right\rangle$



Pr. Zistite oblasť definície funkcie $f: y = 2+5\arccos\left(\frac{7x-1}{5}\right)$

Riešenie:

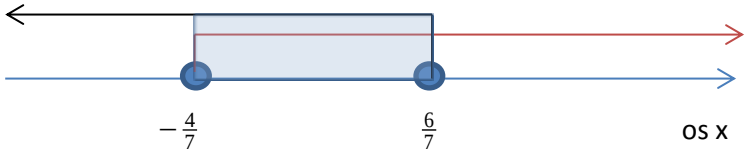
$-1 \le \frac{7x-1}{5}$ a zároveň $\frac{7x-1}{5} \le 1$

$-5 \le 7x-1$ $\quad$ $7x-1 \le 5$

$-4 \le 7x$ $\quad$ $7x \le 6$

$-\frac{4}{7} \le x$ $\quad$ $x \le \frac{6}{7}$

$D(f) = <-\frac{4}{7}, \infty) \cap (-\infty, \frac{6}{7}> \quad \Rightarrow \quad D(f) = \left\langle -\frac{4}{7}, \frac{6}{7} \right\rangle$



Pr. Vyjadrite inverznú funkciu k funkcii $f: y = 1 + 4\arcsin\left(\frac{3x-2}{5}\right)$

Riešenie:

$f^{-1}: \quad x = 1 + 4\arcsin\left(\frac{3y-2}{5}\right)$

$$\frac{x-1}{4} = \arcsin\frac{3y-2}{5}$$

$$\sin\left(\frac{x-1}{4}\right) = \frac{3y-2}{5}$$

$$5\cdot\sin\left(\frac{x-1}{4}\right) = 3y-2$$

$$2+5\cdot\sin\left(\frac{x-1}{4}\right) = 3y$$

$$f^{-1}: \quad y = \frac{2+5\cdot\sin\left(\frac{x-1}{4}\right)}{3}$$

Pr. Vyjadrite inverznú funkciu k funkcii $f: y = 2 + 5\arccos\left(\frac{7x-1}{4}\right)$

Riešenie:

$f^{-1}: \quad x = 2 + 5\arccos\left(\frac{7y-1}{4}\right)$

$$x-2 = 5\arccos\left(\frac{7y-1}{4}\right)$$

$$\frac{x-2}{5} = \arccos\left(\frac{7y-1}{4}\right)$$

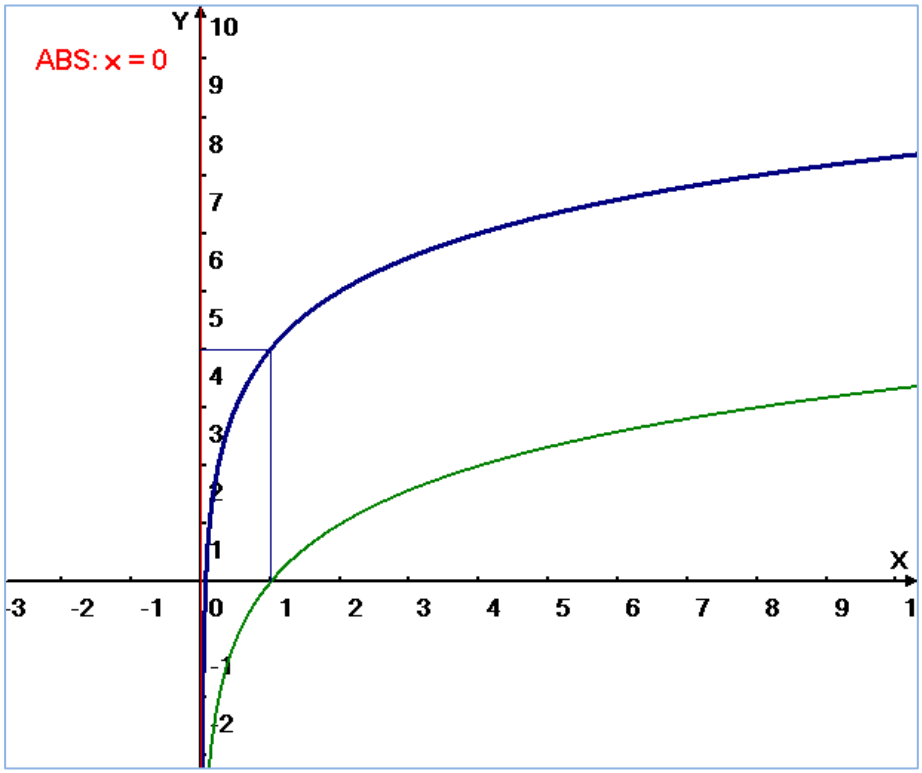
$$\cos\left(\frac{x-2}{5}\right) = \frac{7y-1}{4}$$

$$4\cdot\cos\left(\frac{x-2}{5}\right) = 7y-1$$

$$1+4\cos\left(\frac{x-2}{5}\right) = 7y$$

$$f^{-1}: \quad y = \frac{1+4\cos\left(\frac{x-2}{5}\right)}{7}$$

Pr. Zobrazte graf funkcie $f: y = 4 + \log_2 x$ **a napíšte jej vlastnosti**

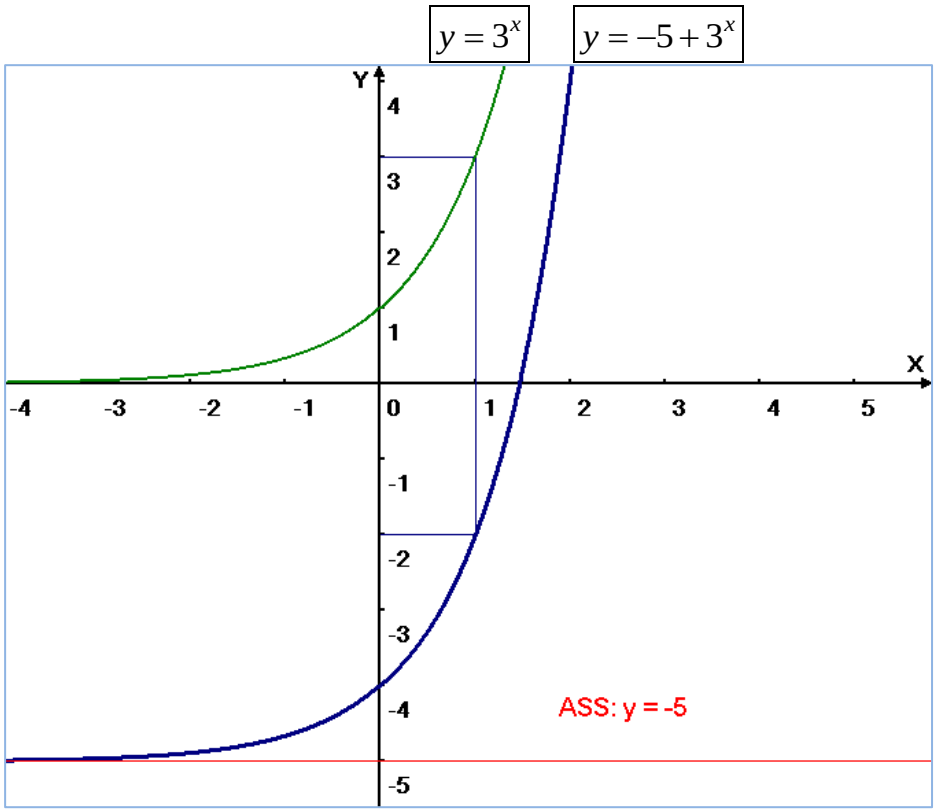


$f: y = 4 + \log_2 x$

$f: y = \log_2 x$

$f: y = 4 + \log_2 x$, **<u>vlastnosti:</u> D(f) = (0, ∞) , H(f) = R, rastúca, prostá, asymptota: x = 0**

Pr. Zobrazte graf funkcie $f: y = -5 + 3^x$ **a napíšte jej vlastnosti**



$f: y = -5 + 3^x$, **<u>vlastnosti:</u> D(f) = R, H(f) = (-5, ∞) , rastúca, prostá, asymptota: y = -5**