

Grafy a vlastnosti funkcií

<https://www.desmos.com/calculator>

LINEÁRNA FUNKCIA $y = ax + b$

Pr.: Zobrazte graf funkcie a napíšte jej vlastnosti

$$f_1: y = 4x - 8$$

$a > 0, a = 4 > 0$? rastúca

významné body: priesečníky s osami

x	0	2
y	-8	0

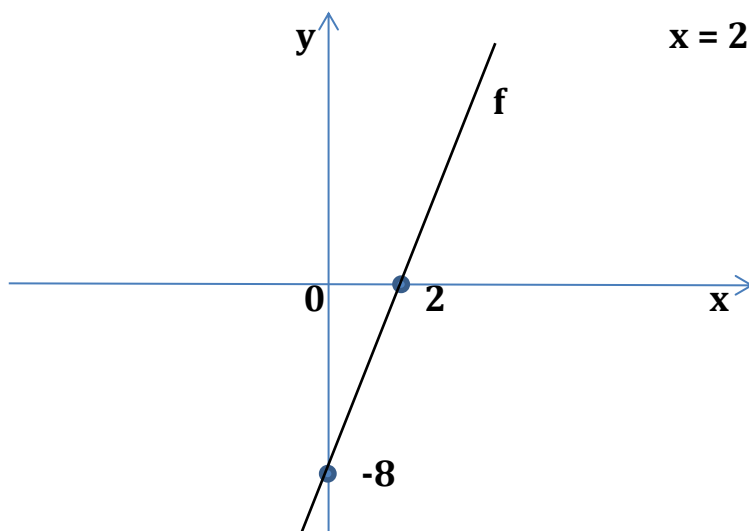
$$x = 0 \text{ ? } y = 4 \cdot 0 - 8 = -8 \quad // \quad y = 0 \text{ ? } 0 = 4x - 8 \text{ (rovnica)}$$

$$8 = 4x$$

$[0, -8], [2, 0]$

$$4x = 8$$

$$x = 2$$



Vlastnosti:

$$D(f) = \mathbb{R}$$

$$H(f) = \mathbb{R}$$

rastúca

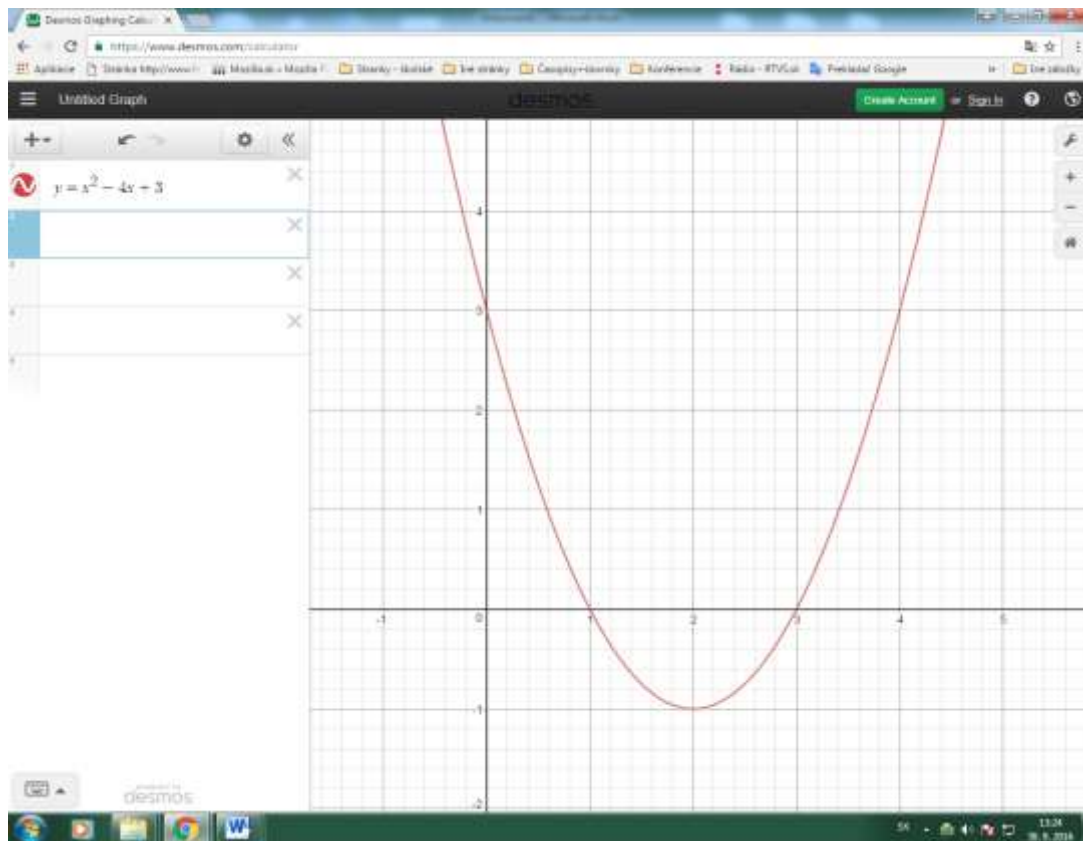
prostá

d.ú. $f_2: y = -3x + 6$

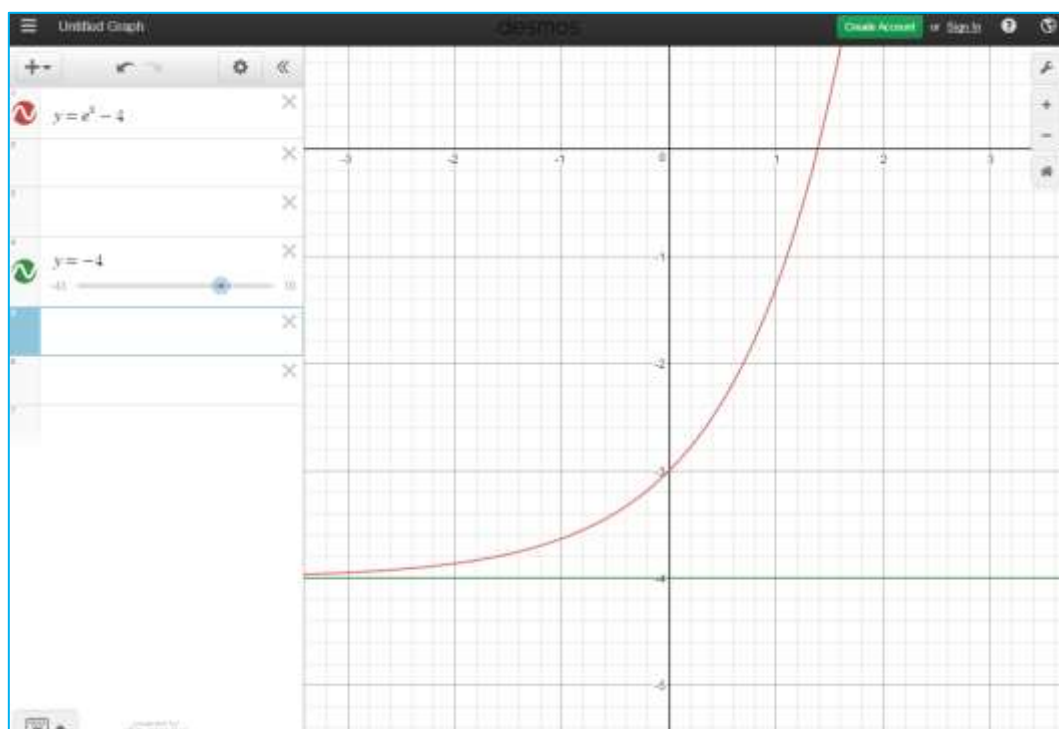
$a < 0, a = -3 < 0$? **klesajúca**

grafy online:: graf <https://www.desmos.com/calculator>

f: $y = x^2 - 4x + 3$



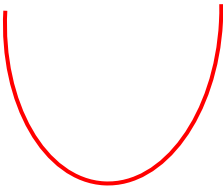
f: $y = e^x - 4$



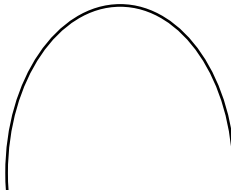
KVADRATICKÁ FUNKCIA

$$f: y = ax^2 + bx + c$$

$a > 0$
konvexná



$a < 0$
konkávna



PR.: Zobraďte graf, napíšte vlastnosti

$$f: y = x^2 - 6x + 5, \quad a = 1, \quad b = -6, \quad c = 5$$

významné body: vrchol paraboly $V = \left[-\frac{b}{2a}, f\left(-\frac{b}{2a}\right) \right]$

$$x = -\frac{-6}{2 \cdot 1} = 3$$

$$y = f\left(-\frac{b}{2a}\right) = f(3) = 3^2 - 6 \cdot 3 + 5 = 9 - 18 + 5 = -4$$

$$V = [3, -4]$$

Priesečníky s osami	x	0	5	1
	y	5	0	

$$f: y = x^2 - 6x + 5$$

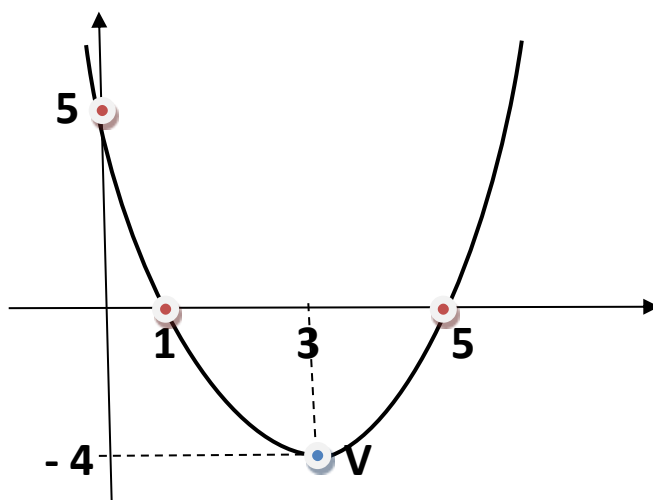
$$x = 0 \Rightarrow y = 0^2 - 6 \cdot 0 + 5 = 5, \quad [0, 5]$$

$$y = 0 \Rightarrow 0 = x^2 - 6x + 5, \quad x_{12} = \frac{-b \pm \sqrt{b^2 - 4ac}}{2a}$$

$$x_{12} = \frac{-(-6) \pm \sqrt{(-6)^2 - 4 \cdot 1 \cdot 5}}{2 \cdot 1} = \frac{6 \pm \sqrt{16}}{2} = \frac{6 \pm 4}{2} =$$

$$x_1 = \frac{10}{2} = 5, \quad x_2 = \frac{2}{2} = 1, \quad [5, 0], \quad [1, 0]$$

Body + parabola: $V = [3, -4]$, $[0, 5]$, $[5, 0]$, $[1, 0]$



Vlastnosti:

$D(f) = \mathbb{R}$, $H(f) = [-4, \infty)$,

ohraničená zdola číslom -4 ,

nie je ani párna ani nepárna,

klesajúca na $(-\infty, 3)$, rastúca na $(3, \infty)$

EXPONENCIÁLNA A LOGARITMICKÁ FUNKCIA

$$y = \log_a x \Leftrightarrow a^y = x$$

PR.: Zobrazte graf, napíšte vlastnosti a zistite inverznú

funkciu $\Leftrightarrow$

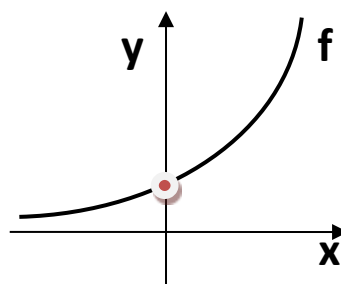
rastúca

$$f : y = 3^x + 4$$

$$y = a^x, a > 1$$

os x asymptota

(graf sa blíži k osi x)



$$f : y = 3^x + 4$$

$a = 3, 3 > 1$, rastúca exp. funkcia

asymptota $y = 4$ (graf sa blíži k priamke $y = 4$)

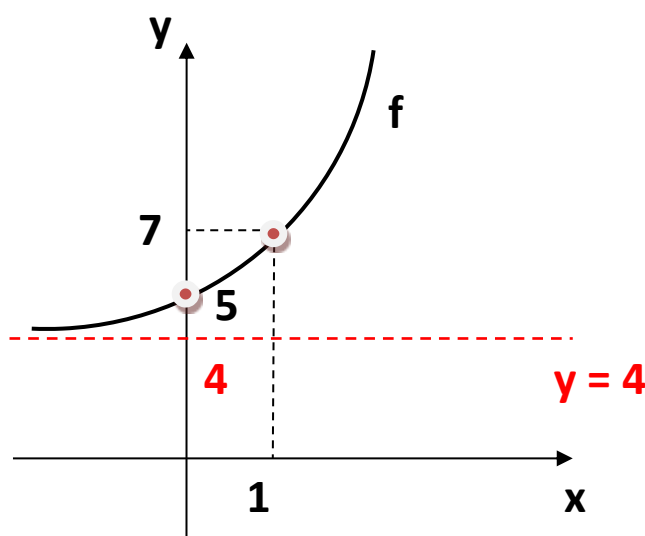
významné body:

x	0	Neex.	1
y	5	0	7

$$x = 0 \Rightarrow y = 3^0 + 4 = 1 + 4 = 5, [0, 5]$$

$$y = 0 \Rightarrow 0 = 3^x + 4$$

$$-4 = 3^x \text{ nemá riešenie, bod neexistuje}$$



Vlastnosti:

$$D(f) = \mathbb{R}, H(f) = (4, \infty)$$

Ohraničená zdola číslom 4

Rastúca, Prostá

Inverzná funkcia

$$f : y = 3^x + 4$$

$$f^{-1} : x = 3^y + 4 \quad / - 4$$

$$x - 4 = 3^y \quad / \circ \log_3()$$

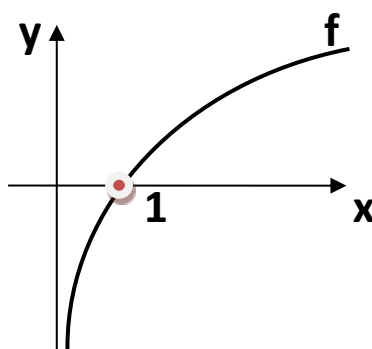
$$\log_3(x - 4) = y$$

$$f^{-1} : y = \log_3(x - 4)$$

$$y = \log_a x, a > 1$$

os y asymptota

(graf sa blíži k osi y)



Graf $f^{-1} : y = \log_3(x - 4)$

$a = 3$, $3 > 1$, rastúca log. funkcia

asymptota $x = 4$ (graf sa blíži k priamke $x = 4$)

$$D(f): x - 4 > 0 \iff x > 4$$

významné body:

$$x = 0 \Rightarrow y = \log_3(0 - 4) = \log_3(-4) \quad \text{!!! neexistuje}$$

$$y = 0 \Rightarrow 0 = \log_3(x - 4)$$

$$3^0 = x - 4$$

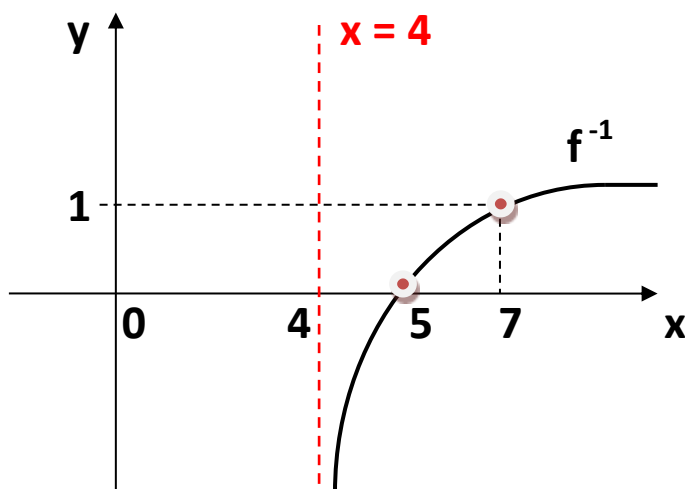
$$1 + 4 = x$$

$$x = 5, \quad [5, 0]$$

$$x = 7 \Rightarrow y = \log_3(7 - 4) = \log_3 3 = 1$$

vzorec !!! $\log_a a = 1$

x	0	5	7
y	Neex.	0	1

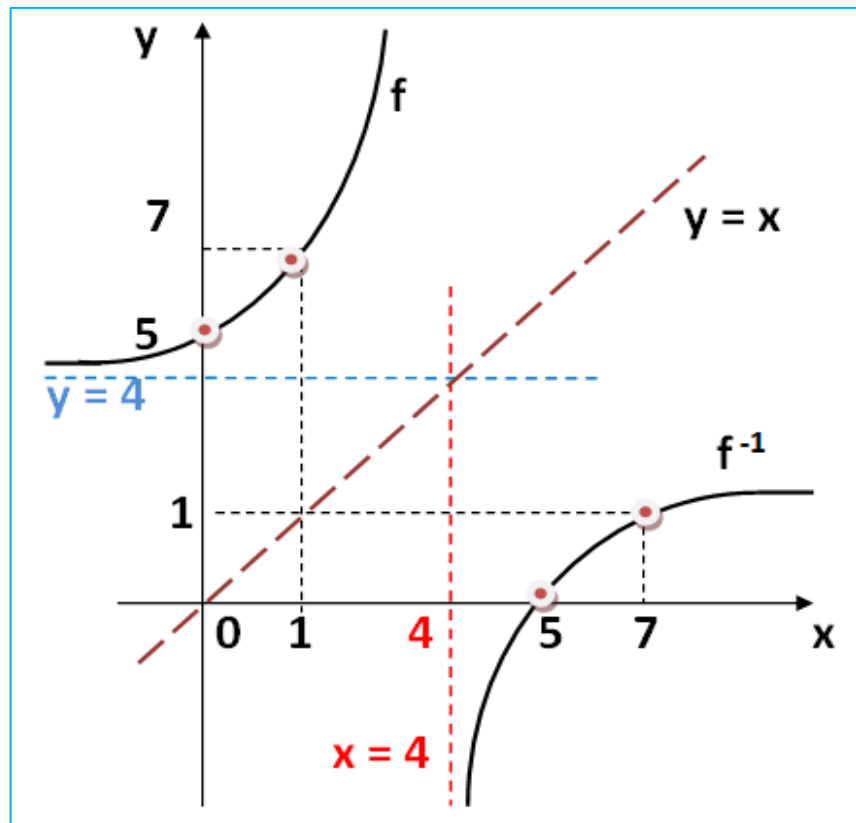


Vlastnosti:

$D(f) = (4, \infty)$, $H(f) = \mathbb{R}$

Nie je ohraničená

Rastúca, Prostá



$$y = \log_{0,1}(x + 2)$$

