Neurčitý integrál (priamo a rozkladom) : 1. téma + návody

1.1. $\int (3x+\frac{1}{x^2})\,dx = \int 3x\,dx + \int \frac{1}{x^2}dx = 3\int x\,dx + \int x^{-2}dx =$....(použite vzorce)

1.2. $\int \frac{8}{x}dx = 8\int \frac{1}{x}dx =$....(vzorec)

1.3. $\int (4\cos x - 5\sin x)dx = 4\int \cos x\,dx - 5\int \sin x\,dx =$... (vzorce)

1.4. $\int (5-\frac{4}{\sqrt{x}})dx = \int 5dx - 4\int \frac{1}{\sqrt{x}}dx = 5\int 1dx - 4\int \frac{1}{x^{\frac{1}{2}}}dx = 5x - 4\int x^{-\frac{1}{2}}dx =$ ...(vzorec)

1.5. $\int (3e^x - 2x - x^2)\,dx = \int 3e^x dx - \int 2x\cdot dx - \int x^2 dx = 3\int e^x dx - 2\int x\cdot dx - \int x^2 dx =$...(vzorce)

1.6. $\int e^x \cdot \left(\frac{1}{e^x} - 3\right)dx = \int (1-3e^x)dx = \int 1dx - \int 3e^x dx = x - 3\int e^x dx =$...(vzorec)

návod: roznásobte zátvorku

1.7. $\int \frac{10}{\sqrt{9-x^2}}dx = 10\int \frac{1}{\sqrt{9-x^2}}dx =$...(vzorce)

1.8. $\int (\frac{4}{\sin^2 x} - \frac{5}{1+x^2})dx = \int \frac{4}{\sin^2 x}dx - \int \frac{5}{1+x^2}dx = 4\int \frac{1}{\sin^2 x}dx - 5\int \frac{1}{1+x^2}dx =$ ...

(vzorce)

1.9. $\int \frac{8}{3x-6}dx = 8\int \frac{1}{3(x-2)}dx = \frac{8}{3}\int \frac{1}{x-2}dx =$...(vzorec)

1.10. $\int \frac{\sin^2 x}{\cos^2 x}dx =$ /návod: platí vzťah $\sin^2 x + \cos^2 x = 1$/

$= \int \frac{1-\cos^2 x}{\cos^2 x}dx = \int (\frac{1}{\cos^2 x} - \frac{\cos^2 x}{\cos^2 x})\,dx = \int \frac{1}{\cos^2 x}dx - \int 1\,dx =$...(vzorce)

Neurčitý integrál (substitúcia): 2a. téma + návody

Vypočítajte neurčitý integrál pomocou vhodnej substitúcie :

2.1. $\int (1-2x)^4\,dx = \int t^4 \frac{dt}{-2} = \frac{-1}{2}\int t^4\,dt = \frac{-1}{2}\cdot\frac{t^5}{5}+c = \frac{-1}{10}\cdot(1-2x)^5+c$

substitúcia: $t = 1-2x \quad\Rightarrow\quad dt=-2dx \quad\Rightarrow\quad \frac{dt}{-2}=dx$

2.2. $\int (2-3x)^5\,dx = \int t^5 \frac{dt}{-3} = \frac{-1}{3}\int t^5\,dt = ...$

substitúcia: $t=2-3x \quad\Rightarrow\quad dt=-3dx \quad\Rightarrow\quad \frac{dt}{-3}=dx$

2.3. $\int \frac{1}{\sqrt{4-2x}}\,dx = \int \frac{1}{\sqrt{t}}\cdot\frac{dt}{-2} = -\frac{1}{2}\int\frac{1}{\sqrt{t}}\cdot dt = -\frac{1}{2}\int\frac{1}{t^{\frac{1}{2}}}\cdot dt = -\frac{1}{2}\int t^{-\frac{1}{2}}\,dt = -\frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}}+c = ...$

substitúcia: $t=4-2x$

2.4. $\int \frac{4x}{\sqrt{x^2-5}}\,dx = \int \frac{4x}{\sqrt{t}}\cdot\frac{dt}{2x} = 2\int\frac{1}{\sqrt{t}}\cdot\frac{dt}{1} = 2\int\frac{1}{\sqrt{t}}\cdot dt = 2\int t^{-\frac{1}{2}}\,dt = ...$

substitúcia: $t=x^2-5 \quad\Rightarrow\quad dt=2xdx \quad\Rightarrow\quad \frac{dt}{2x}=dx$

2.5. $\int \frac{\sin x}{\sqrt{\cos^3 x}}\,dx = \int \frac{\sin x}{\sqrt{t^3}}\cdot\frac{dt}{-\sin x} = -\int\frac{1}{\sqrt{t^3}}\cdot dt = ...$

substitúcia: $t=\cos x \quad\Rightarrow\quad dt=-\sin x\,dx \quad\Rightarrow\quad \frac{dt}{-\sin x}=dx$

2.6. $\int (e^{4x}-\cos 3x)dx = \int e^{4x}dx - \int\cos 3x dx = \int e^t\frac{dt}{4} - \int\cos z\cdot\frac{dz}{3} = ...$

substitúcia I: $t=4x \quad\Rightarrow\quad \frac{dt}{4}=dx$, substitúcia II: $z=3x \quad\Rightarrow\quad \frac{dz}{3}=dx$

2.7. $\int \frac{5-\ln^2 x}{x}\,dx = \int\frac{5}{x}dx - \int\frac{\ln^2 x}{x}dx = 5\int\frac{1}{x}dx - \int\frac{\ln^2 x}{x}dx = 5\ln|x| - \int\frac{t^2}{x}\cdot x\cdot dt = ...$

substitúcia: $t=\ln x \quad\Rightarrow\quad dt=\frac{1}{x}dx \quad\Rightarrow\quad x\cdot dt = dx$

2.8. $\int \frac{\mathrm{arctg}\, x}{1+x^2}\,dx = \int\frac{t}{1+x^2}\cdot(1+x^2)dt = \int t\ dt = ...$

substitúcia: $t = arctgx \quad\Rightarrow\quad dt=\frac{1}{1+x^2}dx \quad\Rightarrow\quad (1+x^2)dt=dx$

Neurčitý integrál (typové): 2b. téma + návody

Vypočítajte neurčitý integrál pomocou typovej substitúcie:

2.9. $\int \frac{1}{x^2+4x+8}dx = \int \frac{1}{(x+2)^2-4+8}dx = \int \frac{1}{(x+2)^2+4}dx = *$

menovateľ upravíme na druhú mocninu dvojčlena a potom urobíme substitúciu : $t = x+2$

$\Rightarrow \ dt = dx$

$* = \int \frac{1}{t^2+4}dt = \frac{1}{2}arctg\frac{t}{2}+c = \frac{1}{2}arctg\frac{x+2}{2}+c$

2.10. $\int \frac{3}{x^2-6x+15}dx = \int \frac{1}{(x-3)^2-9+15}dx = \int \frac{1}{(x-3)^2-9+15}dx = \int \frac{1}{(x-3)^2+6}dx = *$

menovateľ upravíme na druhú mocninu dvojčlena a potom urobíme substitúciu : $t = x-3 \ \Rightarrow \ dt = dx$

$* = \int \frac{1}{t^2+6}dt = \frac{1}{\sqrt{6}}arctg\frac{t}{\sqrt{6}}+c = \frac{1}{\sqrt{6}}arctg\frac{x-3}{\sqrt{6}}+c$

2.11. $\int \frac{3x-1}{x^2+4}dx =$ (rozložíme na dva integrály) $= \int (\frac{3x}{x^2+4} - \frac{1}{x^2+4})dx =$

$= \int \frac{3x}{x^2+4}dx - \int \frac{1}{x^2+4}dx = 3\int \frac{x}{x^2+4}dx - \int \frac{1}{x^2+4}dx = 3\cdot\frac{1}{2}\int \frac{2\cdot x}{x^2+4}dx - \int \frac{1}{x^2+4}dx =$

$= \frac{3}{2}\ln\left|x^2+4\right| - \frac{1}{2}arctg\frac{x}{2}+c$

2.12. $\int \frac{x-4}{\sqrt{x^2+9}}dx =$ (rozložíme na dva integrály) $= \int (\frac{x}{\sqrt{x^2+9}} - \frac{4}{\sqrt{x^2+9}})dx =$

$= \int \frac{x}{\sqrt{x^2+9}}dx - 4\int \frac{1}{\sqrt{x^2+9}}dx = *$ 1.integrál počítame substitučnou metódou,

substitúcia: $t = x^2+9 \Rightarrow dt = 2xdx \ \Rightarrow \ \frac{dt}{2x} = dx$

$* = \int \frac{x}{\sqrt{t}}\cdot\frac{dt}{2x} - 4\int \frac{1}{\sqrt{x^2+9}}dx = \int \frac{1}{\sqrt{t}}\cdot\frac{dt}{2} - 4\int \frac{1}{\sqrt{x^2+9}}dx = \frac{1}{2}\int \frac{1}{\sqrt{t}}\cdot dt - 4\int \frac{1}{\sqrt{x^2+9}}dx =$

$= \frac{1}{2}\int t^{-\frac{1}{2}}\cdot dt - 4\int \frac{1}{\sqrt{x^2+9}}dx = \frac{1}{2}\frac{t^{\frac{1}{2}+1}}{\frac{1}{2}+1} - 4\ln\left|x+\sqrt{x^2+9}\right|+c = \frac{t^{\frac{3}{2}}}{3} - 4\ln\left|x+\sqrt{x^2+9}\right|+c =$

$= \frac{\sqrt{t^3}}{3} - 4\ln\left|x+\sqrt{x^2+9}\right|+c = \frac{\sqrt{(x^2+9)^3}}{3} - 4\ln\left|x+\sqrt{x^2+9}\right|+c$

Neurčitý integrál (per partes): 3. téma + návody

Vypočítajte neurčitý integrál metódou per partes:

3.1. $\int x \cdot \cos x dx = x \cdot \sin x - \int 1 \cdot \sin x dx = x \cdot \sin x - \int \sin x dx = x \cdot \sin x + \cos x + c$

$u = x \quad \Rightarrow \quad u' = 1$

$v' = \cos x \quad \Rightarrow \quad v = \sin x$

3.2. $\int x^2 \cdot e^x dx = x^2 \cdot e^x - \int 2x \cdot e^x dx = x^2 \cdot e^x - (2x \cdot e^x - \int 2 \cdot e^x dx) =$ *

$u = x^2 \quad \Rightarrow \quad u' = 1 \qquad u = 2x \quad \Rightarrow \quad u' = 2$

$v' = e^x \quad \Rightarrow \quad v = e^x \qquad v' = e^x \quad \Rightarrow \quad v = e^x$

* $= x^2 \cdot e^x - 2x \cdot e^x + 2\int e^x dx = x^2 \cdot e^x - 2x \cdot e^x + 2 \cdot e^x + c$

3.3. $\int \frac{\ln x}{x^3} dx = -\frac{1}{2x^2} \cdot \ln x - \int \frac{1}{x} \cdot \left(-\frac{1}{2x^2}\right) dx = -\frac{1}{2x^2} \cdot \ln x + \frac{1}{2}\int \frac{1}{x^3} dx =$

$= -\frac{1}{2x^2} \cdot \ln x - \frac{1}{2}\frac{1}{2x^2} + c = -\frac{1}{2x^2} \cdot \ln x - \frac{1}{4x^2} + c$

$u = \ln x \quad \Rightarrow \quad u' = \frac{1}{x}$

$v' = \frac{1}{x^3} \quad \Rightarrow \quad v = \int \frac{1}{x^3} dx = \int x^{-3} dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

3.4. $\int x \cdot e^{-3x} dx = x \cdot \left(-\frac{1}{3}e^{-3x}\right) - \int 1 \cdot \left(-\frac{1}{3}e^{-3x}\right) dx = -\frac{x \cdot e^{-3x}}{3} + \frac{1}{3}\int e^{-3x} dx =$ *

$u = x \quad \Rightarrow \quad u' = 1$

$v' = e^{-3x} \quad \Rightarrow \quad v = \int e^{-3x} dx = \left| \text{substitúcia } t = -3x \right| = \int e^t \frac{dt}{-3} = \frac{-1}{3}\int e^t dt = -\frac{1}{3}e^{-3x}$

* $= -\frac{x \cdot e^{-3x}}{3} + \frac{1}{3} \cdot \left(-\frac{1}{3}e^{-3x}\right) + c = -\frac{x \cdot e^{-3x}}{3} - \frac{1}{9} \cdot e^{-3x} + c$

3.5. $\int (3 + \ln x) dx = \int 3 dx + \int 1 \cdot \ln x dx = 3x + (x \cdot \ln x - \int \frac{1}{x} \cdot x dx) = 3x + x \cdot \ln x - \int 1 dx =$ *

$u = \ln x \quad \Rightarrow \quad u' = \frac{1}{x}$

$v' = 1 \quad \Rightarrow \quad v = x$

* $= 3x + x \cdot \ln x - x + c = 2x + x \cdot \ln x + c$

3.6. $\int arctgx dx = x \cdot arctgx - \int \frac{1}{x^2+1} \cdot x dx = x \cdot arctgx - \int \frac{x}{x^2+1} dx =$ *

$u = arctgx \Rightarrow \quad u' = \frac{1}{x^2+1}$

$v' = 1 \quad \Rightarrow \quad v = x$

* $= x \cdot arctgx - \frac{1}{2}\int \frac{2x}{x^2+1} dx = x \cdot arctgx - \frac{1}{2}\ln\left|x^2+1\right| + c$ 1.4