## **Neurčitý integrál (goniometrické funkcie): 4. téma + návody**

Vypočítajte neurčitý integrál z goniometrických funkcií:

**4.4.** $$\int \cos^3 x \cdot \sin x\, dx = \int t^3 \cdot \sin x \cdot \frac{1}{-\sin x}\, dx = -\int t^3 dx = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$$

3-nepárny , 1- nepárny, 3>1 $\Rightarrow$ substitúcia : $t = \cos x$, $dt = -\sin x dx$, $\dfrac{dt}{-\sin x} = dx$

**4.5.** $$\int \cos^5 x \cdot \sin^5 x\, dx = \text{/upravíme fukciu/} = \int \cos x \cdot \cos^4 x \cdot \sin^5 x\, dx = *$$

5-nepárny, 5- nepárny (môžeme robiť súbstitúciu $t = \cos x$ alebo $t = \sin x$)

substitúcia : $t = \sin x$, $dt = \cos x dx$, $\dfrac{dt}{\cos x} = dx$

$$* = \int \cos x \cdot (\cos^2 x)^2 \cdot \sin^5 x\, dx = \int \cos x \cdot (1-\sin^2 x)^2 \cdot \sin^5 x\, dx = \text{/teraz urobíme substitúciu/}$$

$$= \int \cos x \cdot (1-t^2)^2 \cdot t^5 \cdot \frac{dt}{\cos x} = \int (1-t^2)^2 \cdot t^5 \cdot dt = \int (1-t^2)^2 \cdot t^5 \cdot dt = \int (1-2t^2+t^4) \cdot t^5 \cdot dt =$$

$$= \int (t^5 - 2t^7 + t^9)\, dt = \frac{t^6}{6} - 2\frac{t^8}{8} + \frac{t^{10}}{10} + c = \frac{\sin^6 x}{6} - \frac{\sin^8 x}{4} + \frac{\sin^{10} x}{10} + c$$

**4.6.** $$\int \cos^2 x \cdot \sin^3 x\, dx = \int \cos^2 x \cdot \sin x \cdot \sin^2 x\, dx = \int \cos^2 x \cdot \sin x \cdot (1-\cos^2 x)\, dx = *$$

2-párny , 3- nepárny $\Rightarrow$ substitúcia : $t = \cos x \Rightarrow \dfrac{dt}{-\sin x} = dx$

$$* = \int t^2 \cdot \sin x \cdot (1-t^2) \frac{dt}{-\sin x} = -\int t^2 \cdot (1-t^2)\, dt = -\int (t^2 - t^4)\, dt = -\frac{t^3}{3} + \frac{t^5}{5} + c =$$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$$

**4.7.** $$\int \cos x \cdot \sin^4 x\, dx = \int \cos x \cdot t^4 \cdot \frac{dt}{\cos x} = \int t^4 \cdot dt = \frac{t^5}{5} + c = \frac{\sin^5 x}{5} + c$$

1- nepárny, 4-párny $\Rightarrow$ substitúcia : $t = \sin x \Rightarrow \dfrac{dt}{\cos x} = dx$

**4.8.** $\int \cos^3 x\,dx = \int \cos x \cdot \cos^2 x\,dx = \int \cos x \cdot (1-\sin^2 x)\,dx = \int \cos x \cdot (1-t^2)\cdot \frac{dt}{\cos x} = *$

3- nepárny, 0-párny $\Rightarrow$ substitúcia : $t = \sin x \Rightarrow \frac{dt}{\cos x} = dx$

$* = \int (1-t^2)\cdot dt = t - \frac{t^3}{3} + c = \sin x - \frac{\sin^3 x}{3} + c$

**4.9.** $\int \cos^2 4x\,dx = \int \frac{1}{2}(1+\cos 8x)\,dx = \frac{1}{2}\int (1+\cos 8x)\,dx = \frac{1}{2}\int 1\,dx + \frac{1}{2}\int \cos 8x\,dx = *$

2- párny, 0-párny $\Rightarrow$ vzorce

$* = \frac{1}{2}x + \frac{1}{2}\cdot\frac{\sin 8x}{8} + c = \frac{1}{2}x + \frac{\sin 8x}{16} + c$

**4.10.** $\int \cos^2 x \cdot \sin^2 x\,dx = \int \frac{1}{2}(1+\cos 2x)\cdot\frac{1}{2}(1-\cos 2x)\,dx = \frac{1}{4}\int (1+\cos 2x)\cdot(1-\cos 2x)\,dx = *$

2- párny, 2-párny $\Rightarrow$ vzorce

$* = \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int 1\,dx - \frac{1}{4}\int \cos^2 2x\,dx = \quad \frac{1}{4}x - \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)\,dx =$

2- párny, 0-párny $\Rightarrow$ vzorce

$= \frac{1}{4}x - \frac{1}{8}\int 1\,dx - \frac{1}{8}\int \cos 4x\,dx = \frac{x}{4} - \frac{x}{8} - \frac{1}{8}\cdot\frac{\sin 4x}{4} + c = \frac{x}{8} - \frac{\sin 4x}{32} + c$

## Neurčitý integrál (rozklad na parciálne zlomky): 5. téma + návody

**5.1.** $\int \frac{x^5}{x^2+2}\,dx =$

Vypočítajte neurčitý integrál **metódou rozkladu na parciálne zlomky**:

**5.2.** $\int \frac{2x-9}{x^2-5x-6}\,dx = \int \frac{2x-9}{(x-6)(x+1)}\,dx = \int \left(\frac{A}{x-6} + \frac{B}{x+1}\right)dx = \int \frac{A}{x-6}\,dx + \int \frac{B}{x+1}\,dx = *$

Menovateľ rozložíme na súčin $(x-6)(x+1)$, následne urobíme rozklad na parciálne zlomky

$$\frac{2x-9}{x^2-5x-6} = \frac{A}{x-6} + \frac{B}{x+1}$$

$$\frac{2x-9}{(x-6)(x+1)} = \frac{A}{x-6} + \frac{B}{x+1} \quad / \cdot \quad (x-6)(x+1) \qquad 1.6$$

$$2x-9 = A(x+1)+B(x-6)$$

$$2x-9 = Ax+A+Bx-6B$$

$$2x-9 = x(A+B)+[A-6B] \Rightarrow$$ **2 = *A* + *B*** (porovnali sme koeficienty pri $x$)

**− 9 = *A* − 6*B*** (porovnali sme absolútne členy)

$$\Rightarrow A = 2 - B$$

$$\underline{-9 = 2 - B - 6B} \Rightarrow -11 = -7B \Rightarrow B = \frac{11}{7} \Rightarrow A = 2 - \frac{11}{7} = \frac{3}{7}$$

$$* = \int \frac{\frac{3}{7}}{x-6}\,dx + \int \frac{\frac{11}{7}}{x+1}\,dx = \frac{3}{7}\int \frac{1}{x-6}\,dx + \frac{11}{7}\int \frac{1}{x+1}\,dx = \underline{\underline{\frac{3}{7}\ln|x-6| + \frac{11}{7}\ln|x+1| + c}}$$

**5.3.** $$\int \frac{3x+4}{x^2-8x+15}\,dx = \int \frac{3x+4}{(x-3)(x-5)}\,dx = \int \frac{A}{x-3}\,dx + \int \frac{B}{x-5}\,dx = *$$

Rozložíme menovateľ na súčin

$$\frac{3x+4}{(x-3)(x-5)} = \frac{A}{x-3} + \frac{B}{x-5} \quad / \cdot \ (x-3)(x-5)$$

$$3x+4 = A(x-5)+B(x-3)$$

$$3x+4 = Ax-5A+Bx-3B$$

$$3x+4 = x(A+B)+(-5A-3B) \Rightarrow$$ **3 = *A* + *B***

**4 = −5*A* − 3*B*** $\Rightarrow A = -\frac{13}{2}, \quad B = \frac{19}{2}$

$$* = \int \frac{-\frac{13}{2}}{x-3}\,dx + \int \frac{\frac{19}{2}}{x-5}\,dx = -\frac{13}{2}\int \frac{1}{x-3}\,dx + \frac{19}{2}\int \frac{1}{x-5}\,dx = \underline{\underline{-\frac{13}{2}\ln|x-3| + \frac{19}{2}\ln|x-5| + c}}$$

**5.4.** $$\int \frac{x-5}{x^2+10x+9}\,dx = \int \frac{x-5}{(x+1)(x+9)}\,dx = \int \frac{A}{x+1}\,dx + \int \frac{B}{x+9}\,dx = *$$

$$\frac{x-5}{(x+1)(x+9)} = \frac{A}{x+1} + \frac{B}{x+9} \quad / \cdot \ (x+1)(x+9)$$

$$x-5 = A(x+9)+B(x+1)$$

$$x-5 = Ax+9A+Bx+B$$

$$1 \cdot x-5 = x(A+B)+(9A+B) \Rightarrow$$ **1 = *A* + *B***

**−5 = 9*A* + *B*** $\Rightarrow A = -\frac{3}{4}, \quad B = \frac{7}{4}$

$$* = \int \frac{-\frac{3}{4}}{x+1}\,dx + \int \frac{\frac{7}{4}}{x+9}\,dx = -\frac{3}{4}\int \frac{1}{x+1}\,dx + \frac{7}{4}\int \frac{1}{x+9}\,dx = \underline{\underline{-\frac{3}{4}\ln|x+1| + \frac{7}{4}\ln|x+9| + c}}$$ 1.7

## *** Určitý integrál ***

**6.1.** $\int_1^2 \left(x^2-2x+2\right)dx=\left[\frac{x^3}{3}-2\frac{x^2}{2}+2x\right]_1^2=\left(\frac{2^3}{3}-2^2+2\cdot 2\right)-\left(\frac{1^3}{3}-1^2+2\cdot 1\right)=\frac{8}{3}-\frac{1}{3}+1=\frac{10}{3}$

**6.2.** $\int_0^1 (4-2x)\,dx=\left[4x-2\frac{x^2}{2}\right]_0^1=\left[4x-x^2\right]_0^1=(4\cdot 1-1^2)-(4\cdot 0-0^2)=4-1=3$

Použite substitúciu:

**6.3.** $\int_0^4 \frac{5x}{\sqrt{x^2+9}}\,dx=$

**6.4.** $\int_0^{\frac{\pi}{2}} \sin^4 x\cdot\cos x\,dx=$

**6.5.** $\int_1^e \frac{3+\ln x}{x}\,dx=\ldots=\left[\frac{t^2}{2}\right]_3^4=\frac{16-9}{4}=\frac{7}{2}$

Použite metódu per partes:

**6.6.** $\int_0^{\pi} x\cdot\cos x\,dx=-2$

**6.7.** $\int_0^1 x\cdot e^{4x}\,dx=\frac{3e^4+1}{16}$

**6.8.** $\int_1^e \frac{\ln x}{x^2}\,dx=-\frac{2}{e}+1$

**6.9.** $\int_1^e x^3\cdot\ln x\,dx=\frac{3e^4-3}{16}$

**6.10.** $\int_1^e \frac{\ln x}{x^3}\,dx=-\frac{3}{4e^2}+\frac{1}{4}$