2. téma – návody

Vypočítajte neurčitý integrál pomocou vhodnej substitúcie :

2.1. $\int (1-2x)^4\,dx = \int t^4 \frac{dt}{-2} = \frac{-1}{2}\int t^4\,dt = \frac{-1}{2}\cdot\frac{t^5}{5} + c = \frac{-1}{10}\cdot(1-2x)^5 + c$

substitúcia: $t = 1-2x \quad\Rightarrow\quad dt = -2dx \quad\Rightarrow\quad \frac{dt}{-2} = dx$

2.2. $\int (2-3x)^5\,dx = \int t^5 \frac{dt}{-3} = \frac{-1}{3}\int t^5\,dt = ...$

substitúcia: $t = 2-3x \quad\Rightarrow\quad dt = -3dx \quad\Rightarrow\quad \frac{dt}{-3} = dx$

2.3. $\int \frac{1}{\sqrt{4-2x}}\,dx = \int \frac{1}{\sqrt{t}}\cdot\frac{dt}{-2} = -\frac{1}{2}\int\frac{1}{\sqrt{t}}\cdot dt = -\frac{1}{2}\int\frac{1}{t^{\frac{1}{2}}}\cdot dt = -\frac{1}{2}\int t^{-\frac{1}{2}}\,dt = -\frac{1}{2}\frac{t^{\frac{1}{2}}}{\frac{1}{2}} + c = ...$

substitúcia: $t = 4-2x$

2.4. $\int \frac{4x}{\sqrt{x^2-5}}\,dx = \int\frac{4x}{\sqrt{t}}\cdot\frac{dt}{2x} = 2\int\frac{1}{\sqrt{t}}\cdot\frac{dt}{1} = 2\int\frac{1}{\sqrt{t}}\cdot dt = 2\int t^{-\frac{1}{2}}\,dt = ...$

substitúcia: $t = x^2-5 \quad\Rightarrow\quad dt = 2xdx \quad\Rightarrow\quad \frac{dt}{2x} = dx$

2.5. $\int \frac{\sin x}{\sqrt{\cos^3 x}}\,dx = \int\frac{\sin x}{\sqrt{t^3}}\cdot\frac{dt}{-\sin x} = -\int\frac{1}{\sqrt{t^3}}\cdot dt = ...$

substitúcia: $t = \cos x \quad\Rightarrow\quad dt = -\sin x\,dx \quad\Rightarrow\quad \frac{dt}{-\sin x} = dx$

2.6. $\int (e^{4x} - \cos 3x)dx = \int e^{4x}dx - \int \cos 3x dx = \int e^t \frac{dt}{4} - \int \cos z\cdot\frac{dz}{3} = ...$

substitúcia I: $t = 4x \quad\Rightarrow\quad \frac{dt}{4} = dx$, substitúcia II: $z = 3x \quad\Rightarrow\quad \frac{dz}{3} = dx$

2.7. $\int \frac{5-\ln^2 x}{x}\,dx = \int\frac{5}{x}dx - \int\frac{\ln^2 x}{x}dx = 5\int\frac{1}{x}dx - \int\frac{\ln^2 x}{x}dx = 5\ln|x| - \int\frac{t^2}{x}\cdot x\cdot dt = ...$

substitúcia: $t = \ln x \quad\Rightarrow\quad dt = \frac{1}{x}dx \quad\Rightarrow\quad x\cdot dt = dx$

2.8. $\int \frac{\operatorname{arctg} x}{1+x^2}\,dx = \int\frac{t}{1+x^2}\cdot(1+x^2)dt = \int t\;dt = ...$

substitúcia: $t = arctgx \quad\Rightarrow\quad dt = \frac{1}{1+x^2}dx \quad\Rightarrow\quad (1+x^2)dt = dx$