3. téma – DS – návody

Vypočítajte neurčitý integrál metódou per partes:

3.1. $\int x\cdot\cos x dx = x\cdot\sin x - \int 1\cdot\sin x dx = x\cdot\sin x - \int \sin x dx = x\cdot\sin x + \cos x + c$

$u = x \quad \Rightarrow \quad u' = 1$

$v' = \cos x \quad \Rightarrow \quad v = \sin x$

3.2. $\int x^2\cdot e^x dx = x^2\cdot e^x - \int 2x\cdot e^x dx = x^2\cdot e^x - (2x\cdot e^x - \int 2\cdot e^x dx) = *$

$u = x^2 \quad \Rightarrow \quad u' = 2x \qquad u = 2x \quad \Rightarrow \quad u' = 2$

$v' = e^x \quad \Rightarrow \quad v = e^x \qquad v' = e^x \quad \Rightarrow \quad v = e^x$

$* = x^2\cdot e^x - 2x\cdot e^x + 2\int e^x dx = x^2\cdot e^x - 2x\cdot e^x + 2\cdot e^x + c$

3.3. $\int x^5\cdot\ln x\, dx = (\ln x)\cdot\frac{x^6}{6} - \int \frac{1}{x}\cdot\frac{x^6}{6}dx = \frac{x^6}{6}\ln x - \frac{1}{6}\int x^5 dx = *$

$u = \ln x \quad \Rightarrow \quad u' = \frac{1}{x}$

$v' = x^5 \quad \Rightarrow \quad v = \int x^5 dx = \frac{x^6}{6}$

$* = \frac{x^6}{6}\ln x - \frac{1}{6}\cdot\frac{x^6}{6} + \text{c} = \frac{x^6}{6}(\ln x - \frac{1}{6}) + \text{c}$

3.4. $\int \frac{\ln x}{x^3}dx = -\frac{1}{2x^2}\cdot\ln x - \int \frac{1}{x}\cdot\left(-\frac{1}{2x^2}\right)dx = -\frac{1}{2x^2}\cdot\ln x + \frac{1}{2}\int\frac{1}{x^3}dx =$

$= -\frac{1}{2x^2}\cdot\ln x - \frac{1}{2}\frac{1}{2x^2} + c = -\frac{1}{2x^2}\cdot\ln x - \frac{1}{4x^2} + c$

$u = \ln x \quad \Rightarrow \quad u' = \frac{1}{x}$

$v' = \frac{1}{x^3} \quad \Rightarrow \quad v = \int\frac{1}{x^3}dx = \int x^{-3}dx = \frac{x^{-2}}{-2} = -\frac{1}{2x^2}$

3.5. $\int x\cdot e^{-3x}dx = x\cdot\left(-\frac{1}{3}e^{-3x}\right) - \int 1\cdot\left(-\frac{1}{3}e^{-3x}\right)dx = -\frac{x\cdot e^{-3x}}{3} + \frac{1}{3}\int e^{-3x}dx = *$

$u = x \quad \Rightarrow \quad u' = 1$

$v' = e^{-3x} \quad \Rightarrow \quad v = \int e^{-3x}dx = \left|\text{substitúcia } t = -3x\right| = \int e^t\frac{dt}{-3} = \frac{-1}{3}\int e^t dt = -\frac{1}{3}e^{-3x}$

$* = -\frac{x\cdot e^{-3x}}{3} + \frac{1}{3}\cdot\left(-\frac{1}{3}e^{-3x}\right) + c = -\frac{x\cdot e^{-3x}}{3} - \frac{1}{9}\cdot e^{-3x} + c$

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3.6. $\int (3+\ln x)dx = \int 3dx + \int 1\cdot \ln x dx = 3x + (x\cdot \ln x - \int \frac{1}{x}\cdot x dx) = 3x + x\cdot \ln x - \int 1 dx =$ *

$u = \ln x \Rightarrow u' = \frac{1}{x}$

$v' = 1 \Rightarrow v = x$

* $= 3x + x\cdot \ln x - x + c = 2x + x\cdot \ln x + c$

3.7. $\int arctgx\, dx =$ * $u = arctgx \Rightarrow u' = \frac{1}{x^2+1}$ * $= x\cdot arctgx - \int \frac{1}{x^2+1}\cdot x\, dx$

* $v' = 1 \Rightarrow v = x$ *

$= x\cdot arctgx - \int \frac{x}{x^2+1}dx = x\cdot arctgx - \frac{1}{2}\int \frac{2x}{x^2+1}dx = x\cdot arctgx - \frac{1}{2}\ln\left|x^2+1\right| + c$

3.8. $\int \arcsin x\, dx = \int 1\cdot \arcsin x\, dx =$ * $u = \arcsin x \Rightarrow u' = \frac{1}{\sqrt{1-x^2}}$

$v' = 1 \Rightarrow v = x$ *

$= x\cdot \arcsin x - \int \frac{x}{\sqrt{1-x^2}}dx =$ * substitúcia $t = 1 - x^2$ *.....$= x\cdot \arcsin x + \sqrt{1-x^2} + c$

3.9. $\int e^{-x}\cdot \sin x\, dx = e^{-x}\cdot(-\cos x) - \int -e^{-x}\cdot(-\cos x)\, dx = -e^{-x}\cdot \cos x - \int e^{-x}\cdot \cos x\, dx =$ *

$u = e^{-x} \Rightarrow u' = -e^{-x}$ | $u = e^{-x} \Rightarrow u' = -e^{-x}$

$v' = \sin x \Rightarrow v = -\cos x$ | $v' = \cos x \Rightarrow v = \sin x$

* $= -e^{-x}\cdot \cos x - (e^{-x}\cdot \sin x - \int -e^{-x}\cdot \sin x\, dx) = -e^{-x}\cdot \cos x - e^{-x}\cdot \sin x - \int e^{-x}\cdot \sin x\, dx$

$\Rightarrow \int e^{-x}\cdot \sin x\, dx = -e^{-x}\cdot \cos x - e^{-x}\cdot \sin x - \int e^{-x}\cdot \sin x\, dx \quad // + \int e^{-x}\cdot \sin x\, dx$

$2\int e^{-x}\cdot \sin x\, dx = -e^{-x}\cdot \cos x - e^{-x}\cdot \sin x \quad // \cdot \frac{1}{2}$

$\int e^{-x}\cdot \sin x\, dx = \frac{-e^{-x}\cdot \cos x - e^{-x}\cdot \sin x}{2} + c$

3.10. $\int \frac{x^2}{(x^2+5)^2}dx = \int x\cdot \frac{x}{(x^2+5)^2}dx = x\cdot\left(-\frac{1}{2(x^2+5)}\right) - \int 1\cdot(-\frac{1}{2(x^2+5)})dx =$

* $u = x \Rightarrow u' = 1$

$v' = \frac{x}{(x^2+5)^2} \Rightarrow v = \int \frac{x}{(x^2+5)^2}dx = \int \frac{x}{t^2}\cdot\frac{dt}{2x} = \frac{1}{2}\int \frac{1}{t^2}\cdot dt = \frac{1}{2}\cdot\frac{t^{-1}}{-1} = -\frac{1}{2(x^2+5)}$

* substitúcia $t = x^2+5 \Rightarrow dt = 2xdx \Rightarrow \frac{dt}{2x} = dx$ *

$= -\frac{x}{2(x^2+5)} + \frac{1}{2}\int \frac{1}{x^2+5}dx = -\frac{x}{2(x^2+5)} + \frac{1}{2}\cdot\frac{1}{\sqrt{5}}\cdot arctg\frac{x}{\sqrt{5}} + c$