5. téma – návody

Vypočítajte neurčitý integrál **metódou rozkladu na parciálne zlomky** (alebo vydelením):

5.0. a) $\int \frac{x^3+2x-4}{x+2}dx =$ *

vydelíme mnohočlen z čitateľa menovateľom:

$$(x^3+2x-4):(x+2) = x^2-2x+6+\frac{-16}{x+2}$$

$$\begin{array}{l} -(x^3+2x^2) \\ \hline 0 \quad -2x^2+2x-4 \\ \quad -(-2x^2-4x) \\ \hline \qquad 0+6x-4 \\ \qquad -(6x+12) \\ \hline \qquad\quad -16 \end{array}$$

$$* = \int (x^2-2x+6+\frac{-16}{x+2})dx = \int x^2 dx - 2\int x\,dx + 6\int 1dx - 16\int \frac{1}{x+2}dx =$$

$$= \frac{x^3}{3} - 2\frac{x^2}{2} + 6x - 16\ln|x+2| + c = \frac{x^3}{3} - x^2 + 6x - 16\ln|x+2| + c$$

5.0. b) $\int \frac{x^4+3x^3+9x^2+27x+4}{x^2+9}dx =$ *

vydelíme čitateľa menovateľom:

$$(x^4+3x^3+9x^2+27x+4):(x^2+9) = x^2+3x+\frac{4}{x^2+9}$$

$$* = \int (x^2+3x+\frac{4}{x^2+9})dx = \frac{x^3}{3} + 3\frac{x^2}{2} + 4\frac{1}{3}arctg\frac{x}{3} + c$$

5.1. $\int \frac{2x-9}{x^2-5x-6}dx = \int \frac{2x-9}{(x-6)(x+1)}dx = \int (\frac{A}{x-6}+\frac{B}{x+1})dx = \int \frac{A}{x-6}dx + \int \frac{B}{x+1}dx = *$

Menovateľ rozložíme na súčin $(x-6)(x+1)$, následne urobíme rozklad na parciálne zlomky

$$\frac{2x-9}{x^2-5x-6} = \frac{A}{x-6}+\frac{B}{x+1}$$

$$\frac{2x-9}{(x-6)(x+1)} = \frac{A}{x-6}+\frac{B}{x+1} \quad / \cdot \ (x-6)(x+1)$$

$$2x-9 = A(x+1)+B(x-6)$$

$$2x-9 = Ax+A+Bx-6B$$

$$2x-9 = x(A+B)+[A-6B] \Rightarrow$$ **$2 = A + B$** (porovnali sme koeficienty pri x)

$-9 = A - 6B$ (porovnali sme absolútne členy)

$$\Rightarrow A = 2 - B$$

$$\underline{-9 = 2 - B - 6B} \Rightarrow -11 = -7B \Rightarrow B = \frac{11}{7}$$

$$\Rightarrow A = 2 - \frac{11}{7} = \frac{3}{7}$$

$$* = \int \frac{\frac{3}{7}}{x-6}dx + \int \frac{\frac{11}{7}}{x+1}dx = \frac{3}{7}\int \frac{1}{x-6}dx + \frac{11}{7}\int \frac{1}{x+1}dx = \underline{\underline{\frac{3}{7}\ln|x-6| + \frac{11}{7}\ln|x+1| + c}}$$

5.2. $\int \frac{3x+4}{x^2-8x+15}dx = \int \frac{3x+4}{(x-3)(x-5)}dx = \int \frac{A}{x-3}dx + \int \frac{B}{x-5}dx = *$

Rozložíme menovateľ na súčin:

$$\frac{3x+4}{(x-3)(x-5)} = \frac{A}{x-3}+\frac{B}{x-5} \quad / \cdot \ (x-3)(x-5)$$

$$3x+4 = A(x-5)+B(x-3)$$

$$3x+4 = Ax-5A+Bx-3B$$

$$3x+4 = x(A+B)+(-5A-3B) \Rightarrow$$ **$3 = A + B$**

$4 = -5A - 3B$ $\Rightarrow A = -\frac{13}{2}, \quad B = \frac{19}{2}$

$$* = \int \frac{-\frac{13}{2}}{x-3}dx + \int \frac{\frac{19}{2}}{x-5}dx = -\frac{13}{2}\int \frac{1}{x-3}dx + \frac{19}{2}\int \frac{1}{x-5}dx = \underline{\underline{-\frac{13}{2}\ln|x-3| + \frac{19}{2}\ln|x-5| + c}}$$

5.3. $\int \frac{x-5}{x^2+10x+9}dx = \int \frac{x-5}{(x+1)(x+9)}dx = \int \frac{A}{x+1}dx + \int \frac{B}{x+9}dx =$ *

$$\frac{x-5}{(x+1)(x+9)} = \frac{A}{x+1} + \frac{B}{x+9} \quad / \cdot \ (x+1)\,(x+9)$$

$$x-5 = A(x+9)+B(x+1)$$

$$x-5 = Ax+9A+Bx+B$$

$$1\cdot x-5 = x(A+B)+(9A+B) \Rightarrow$$ **1 = *A* + *B***

–5 = 9*A* + *B* $\Rightarrow A = -\frac{3}{4}, \quad B = \frac{7}{4}$

$$* = \int \frac{-\frac{3}{4}}{x+1}dx + \int \frac{\frac{7}{4}}{x+9}dx = -\frac{3}{4}\int \frac{1}{x+1}dx + \frac{7}{4}\int \frac{1}{x+9}dx = \underline{\underline{-\frac{3}{4}\ln|x+1| + \frac{7}{4}\ln|x+9| + c}}$$

5.4. $\int \frac{x^2+12x+14}{(x-2)(x^2+10)}dx = \int \left(\frac{A}{x-2} + \frac{Bx+C}{x^2+10}\right)dx =$ ¤

$$\frac{A}{x-2} + \frac{Bx+C}{x^2+10} = \frac{x^2+12x+14}{(x-2)(x^2+10)}$$

$$\frac{A(x^2+10)+(Bx+C)(x-2)}{(x-2)(x^2+10)} = \frac{x^2+12x+14}{(x-2)(x^2+10)} \quad /\cdot(x-2)(x^2+10)$$

$$A(x^2+10)+(Bx+C)(x-2) = x^2+12x+14$$

$$Ax^2+10A+Bx^2-2Bx+Cx-2C = x^2+12x+14$$

$$Ax^2+Bx^2-2Bx+Cx+10A-2C = x^2+12x+14$$

$$x^2(A+B)+x(-2B+C)+(+10A-2C) = 1\cdot x^2+12x+14$$

$\Rightarrow$ ***A* + *B* = 1** (porovnali sme koeficienty pri x^2)

–2*B* + *C* = 12 (porovnali sme koeficienty pri x)

10*A* – 2*C* = 14 (porovnali sme absolútne členy)

$\Rightarrow$ ***A* = 3, *B* = – 2, *C* = 8** (riešenie sústavy lineárnych rovníc zistíme, napr. dosadzovacou metódou)

$$¤ \int \left(\frac{A}{x-2} + \frac{Bx+C}{x^2+10}\right)dx = \int \frac{3}{x-2}dx + \int \frac{-2x+8}{x^2+10}dx =$$

$$= 3\ln|x-2| - \int \frac{2x}{x^2+10}dx + 8\int \frac{1}{x^2+10}dx = 3\ln|x-2| - \ln|x^2+10| + 8\frac{8}{\sqrt{10}}arctg\frac{x}{\sqrt{10}} + c$$

5.5. $\int \frac{8x^2+6x+21}{(x+4)(x^2+9)}dx = \int (\frac{A}{x+4}+\frac{Bx+C}{x^2+9})dx =$ ¤

$$\frac{A}{x+4}+\frac{Bx+C}{x^2+9} = \frac{8x^2+6x+21}{(x+4)(x^2+9)}$$

$$\frac{A(x^2+9)+(Bx+C)(x+4)}{(x+4)(x^2+9)} = \frac{8x^2+6x+21}{(x+4)(x^2+9)} \quad / \cdot (x-2)(x^2+10)$$

$$A(x^2+9)+(Bx+C)(x+4) = 8x^2+6x+21$$

$$Ax^2+9A+Bx^2+4Bx+Cx+4C = 8x^2+6x+21$$

$$Ax^2+Bx^2+4Bx+Cx+9A+4C = 8x^2+6x+21$$

$$x^2(A+B)+x(4B+C)+(9A+4C) = 8x^2+6x+21$$

$\Rightarrow$ **$A + B = 8$**

$4B + C = 6$

$9A + 4C = 21$

$\Rightarrow$ **$A = 5$, $B = 3$, $C = -6$** (riešenie sústavy lineárnych rovníc)

¤ $\int (\frac{A}{x+4}+\frac{Bx+C}{x^2+9})dx = \int \frac{5}{x+4}dx + \int \frac{3x-6}{x^2+9}dx =$

$= 5\ln|x+4| + 3\int \frac{x}{x^2+9}dx - 6\int \frac{1}{x^2+9}dx = 5\ln|x+4| + 3\ln|x^2+9| - \frac{6}{3}arctg\frac{x}{3}+c =$

$= 5\ln|x+4| + 3\ln|x^2+9| - 2arctg\frac{x}{3}+c$