# 6. téma – návody

Vypočítajte neurčitý integrál z **goniometrických funkcií**:

**6.1.** $\int \cos^3 x \cdot \sin x \, dx = \quad \int t^3 \cdot \sin x \cdot \frac{1}{-\sin x} dx = -\int t^3 dx = -\frac{t^4}{4} + c = -\frac{\cos^4 x}{4} + c$

3-nepárny , 1- nepárny, 3>1 $\Rightarrow$ substitúcia : $t = \cos x$

$$dt = -\sin x dx$$

$$\frac{dt}{-\sin x} = dx$$

**6.2.** $\int \cos^5 x \cdot \sin^5 x \, dx =$ /upravíme fukciu/ = $\int \cos x \cdot \cos^4 x \cdot \sin^5 x \, dx =$ *

5-nepárny, 5- nepárny (môžeme robiť súbstitúciu $t = \cos x$ alebo $t = \sin x$)

substitúcia : $t = \sin x$

$$dt = \cos x dx$$

$$\frac{dt}{\cos x} = dx$$

* $= \int \cos x \cdot (\cos^2 x)^2 \cdot \sin^5 x \, dx = \int \cos x \cdot (1 - \sin^2 x)^2 \cdot \sin^5 x \, dx =$ /teraz urobíme substitúciu/

$$= \int \cos x \cdot (1 - t^2)^2 \cdot t^5 \cdot \frac{dt}{\cos x} = \int (1 - t^2)^2 \cdot t^5 \cdot dt = \int (1 - 2t^2 + t^4) \cdot t^5 dt =$$

$$= \int (t^5 - 2t^7 + t^9) \, dt = \frac{t^6}{6} - 2\frac{t^8}{8} + \frac{t^{10}}{10} + c = \frac{\sin^6 x}{6} - \frac{\sin^8 x}{4} + \frac{\sin^{10} x}{10} + c$$

**6.3.** $\int \cos^2 x \cdot \sin^3 x \, dx = \quad \int \cos^2 x \cdot \sin x \cdot \sin^2 x \, dx = \quad \int \cos^2 x \cdot \sin x \cdot (1 - \cos^2 x) \, dx =$ *

exponenty sú: 2-párny , 3- nepárny $\Rightarrow$ substitúcia : $t = \cos x \Rightarrow \frac{dt}{-\sin x} = dx$

* $= \int t^2 \cdot \sin x \cdot (1 - t^2) \frac{dt}{-\sin x} = -\int t^2 \cdot (1 - t^2) \, dt = -\int (t^2 - t^4) \, dt = -\frac{t^3}{3} + \frac{t^5}{5} + c =$

$$= -\frac{\cos^3 x}{3} + \frac{\cos^5 x}{5} + c$$

**6.4.** $\int \cos x \cdot \sin^4 x\,dx = \int \cos x \cdot t^4 \cdot \frac{dt}{\cos x} = \int t^4 \cdot dt = \frac{t^5}{5} + c = \underline{\underline{\frac{\sin^5 x}{5} + c}}$

1- nepárny, [4-párny] $\Rightarrow$ substitúcia : $t = \sin x \Rightarrow \frac{dt}{\cos x} = dx$

**6.5.** $\int \cos^3 x\,dx = \int \cos x \cdot \cos^2 x\,dx = \int \cos x \cdot (1-\sin^2 x)\,dx = \int \cos x \cdot (1-t^2) \cdot \frac{dt}{\cos x} =$ *

3- nepárny, [0-párny] $\Rightarrow$ substitúcia : $t = \sin x \Rightarrow \frac{dt}{\cos x} = dx$

* $= \int (1-t^2) \cdot dt = t - \frac{t^3}{3} + c = \underline{\underline{\sin x - \frac{\sin^3 x}{3} + c}}$

**6.6.** $\int \cos^2 4x\,dx = \int \cos^2 t \cdot \frac{dt}{4} = \frac{1}{4}\int \cos^2 t \cdot dt =$ *

substitúcia : $t = 4x \Rightarrow dt = 4dx \Rightarrow \frac{dt}{4} = dx$

2- párny, 0-párny

$\Rightarrow$ vzorec $\boxed{\cos^2(t) = \frac{1}{2}(1+\cos 2t)}$

* $= \frac{1}{4}\int \frac{1}{2}(1+\cos 2t)\,dt = \frac{1}{8}\int (1+\cos 2t)\,dt = \frac{1}{8}\int 1dt + \frac{1}{8}\int \cos 2t\,dt =$ **

substitúcia : $z = 2t \Rightarrow dz = 2dt \Rightarrow \frac{dz}{2} = dt$

** $= \frac{1}{8}t + \frac{1}{8}\int \cos z \cdot \frac{dz}{2} = \frac{1}{8}t + \frac{1}{16}\int \cos z \cdot dz = \frac{1}{8}t + \frac{1}{16}\sin z + c = \frac{1}{8}4x + \frac{1}{16}\sin(2t) + c =$

$= \frac{1}{2}x + \frac{1}{16}\sin(2 \cdot 4x) + c = \underline{\underline{\frac{x}{2} + \frac{\sin 8x}{16} + c}}$

**6.7.** $\int \cos^2 x \cdot \sin^2 x\,dx = \int \frac{1}{2}(1+\cos 2x) \cdot \frac{1}{2}(1-\cos 2x)\,dx = \frac{1}{4}\int (1+\cos 2x)\cdot(1-\cos 2x)\,dx =$ *

2- párny, 2-párny $\Rightarrow$ vzorce $\boxed{\cos^2 x = \frac{1}{2}(1+\cos 2x)}$ $\boxed{\sin^2 x = \frac{1}{2}(1-\cos 2x)}$

* $= \frac{1}{4}\int (1-\cos^2 2x)\,dx = \frac{1}{4}\int 1dx - \frac{1}{4}\int \cos^2 2x\,dx = \frac{1}{4}x - \frac{1}{4}\int \frac{1}{2}(1+\cos 4x)dx =$

2- párny, 0-párny $\Rightarrow$ vzorec $\boxed{\cos^2(kx) = \frac{1}{2}(1+\cos 2kx)}$

$= \frac{1}{4}x - \frac{1}{8}\int 1dx - \frac{1}{8}\int \cos 4x\,dx = \frac{x}{4} - \frac{x}{8} - \frac{1}{8}\cdot\frac{\sin 4x}{4} + c = \underline{\underline{\frac{x}{8} - \frac{\sin 4x}{32} + c}}$