

VEKTORY A MATICE - CVIČENIA

1. V priestore V_4 sú dané vektory $\vec{c} = (-4, 0, 2, 1)$, $\vec{d} = (-3, -8, -6, 2)$. Vypočítajte:

- a) vektor $\vec{v} = \vec{d} - \vec{c}$, vektor $\vec{w} = -3 \cdot \vec{d}$ [$\vec{v} = (1, -8, -8, 1)$, $\vec{w} = (9, 24, 18, -6)$]
 b) vektor $\vec{x} = 2 \cdot \vec{c} + 3 \cdot \vec{d}$ [$\vec{x} = (-17, -24, -14, 8)$]
 c) vektor $\vec{x} = 4(\vec{c} - \vec{d})$ [$\vec{x} = (-4, 32, 32, -4)$]
 d) vypočítajte skalárny súčin $\vec{c} \cdot \vec{d}$ [2]

2. Vyjadrite vektor $\vec{d}$ ako lineárnu kombináciu vektorov $\vec{a}$, $\vec{b}$, $\vec{c}$:

- a) $\vec{d} = (-3, -17, 7)$, $\vec{a} = (1, 3, 2)$, $\vec{b} = (0, 1, -1)$, $\vec{c} = (-1, -4, 0)$ [$\vec{d} = 2\vec{a} - 3\vec{b} + 5\vec{c}$]
 b) $\vec{d} = (11, 4, 30)$, $\vec{a} = (3, 2, 6)$, $\vec{b} = (1, 2, 1)$, $\vec{c} = (2, 2, -2)$ [$\vec{d} = 5\vec{a} - 2\vec{b} - \vec{c}$]
 c) $\vec{d} = (-2, 2, -8)$, $\vec{a} = (1, 3, 1)$, $\vec{b} = (0, 1, 2)$, $\vec{c} = (1, 0, 6)$ [$\vec{d} = 2\vec{b} - 2\vec{c}$]
 d) $\vec{d} = (3, -5, -1)$, $\vec{a} = (2, 1, 1)$, $\vec{b} = (3, 3, 0)$, $\vec{c} = (-1, 2, -3)$

[sústava nemá riešenie, vektor $\vec{d}$ nie je možné vyjadriť ako lineárnu kombináciu vektorov $\vec{a}$, $\vec{b}$, $\vec{c}$,]

3. Vyjadrite vektor $\vec{b}$ ako lineárnu kombináciu vektorov $\vec{a}_1, \vec{a}_2, \vec{a}_3, \vec{a}_4$:

- a) $\vec{b} = (1, 2, 4, -2)$, $\vec{a}_1 = (2, 4, 8, 2)$, $\vec{a}_2 = (-4, -4, -6, 4)$, $\vec{a}_3 = (2, 3, 5, 1)$, $\vec{a}_4 = (-1, -1, -3, 1)$
 b) $\vec{b} = (8, -6, 5, -9)$, $\vec{a}_1 = (1, -1, 2, 1)$, $\vec{a}_2 = (4, -3, 6, 1)$, $\vec{a}_3 = (-2, 5, -9, -9)$, $\vec{a}_4 = (1, -2, 6, 7)$
 [a) $\vec{b} = 2\vec{a}_1 - \vec{a}_2 - 3\vec{a}_3 + \vec{a}_4$, b) $\vec{b} = \vec{a}_1 + 2 \cdot \vec{a}_2 - 1 \cdot \vec{a}_3 - 3 \cdot \vec{a}_4$]

4. Zistite, či sú dané vektory lineárne závislé alebo lineárne nezávislé:

- a) $\vec{a}_1 = (2, 3, -4)$, $\vec{a}_2 = (-1, 1, 1)$, $\vec{a}_3 = (1, -5, 1)$ [nezávislé]
 b) $\vec{a}_1 = (2, 1, 0)$, $\vec{a}_2 = (3, 4, 4)$, $\vec{a}_3 = (-1, 2, -2)$ [nezávislé]
 c) $\vec{a}_1 = (2, 4, 6)$, $\vec{a}_2 = (6, 10, 10)$, $\vec{a}_3 = (-2, -2, 2)$ [závislé]
 d) $\vec{a} = (1, 1, 1, 0)$, $\vec{a} = (2, 3, 1, -1)$, $\vec{c} = (1, -1, 3, 0)$, $\vec{d} = (3, 4, 2, -1)$ [závislé]
 e) $\vec{a}_1 = (1, 1, 1, 4)$, $\vec{a}_2 = (2, -3, 1, 2)$, $\vec{a}_3 = (1, -1, 1, -3)$, $\vec{a}_4 = (1, 2, -1, 1)$ [nezávislé]

5. K daným maticiam vypočítajte súčet $A + B$ a matice $C = 3A - 2B$, $D = 4B - 5A$:

$$A = \begin{pmatrix} 2 & 4 \\ -1 & 3 \\ 3 & 5 \end{pmatrix}, B = \begin{pmatrix} 4 & 2 \\ 1 & 3 \\ 2 & 4 \end{pmatrix} \quad [A + B = \begin{pmatrix} 6 & 6 \\ 0 & 6 \\ 5 & 9 \end{pmatrix}, C = \begin{pmatrix} -2 & 8 \\ -5 & 3 \\ 5 & 7 \end{pmatrix}, D = \begin{pmatrix} 6 & -12 \\ 9 & -3 \\ -7 & -9 \end{pmatrix}]$$

6. Vypočítajte súčiny $A \cdot B$, $B \cdot A$:

$$a) A = \begin{pmatrix} -1 & -2 \\ 10 & 5 \end{pmatrix}, B = \begin{pmatrix} 4 & -3 \\ 1 & 2 \end{pmatrix} \quad [A \cdot B = \begin{pmatrix} -6 & -1 \\ 45 & -20 \end{pmatrix}, B \cdot A = \begin{pmatrix} -34 & -23 \\ 19 & 8 \end{pmatrix}]$$

$$\text{b) } A = \begin{pmatrix} 1 & 2 & -3 \\ 5 & 4 & 0 \end{pmatrix}, B = \begin{pmatrix} -2 & 2 \\ 0 & -1 \\ 4 & 3 \end{pmatrix} \quad [A \cdot B = \begin{pmatrix} -14 & -9 \\ -10 & 6 \end{pmatrix}, B \cdot A = \begin{pmatrix} 8 & 4 & 6 \\ -5 & -4 & 0 \\ 19 & 20 & -12 \end{pmatrix}]$$

$$\text{c) } A = \begin{pmatrix} 2 & 4 \\ -1 & 5 \\ 3 & 1 \end{pmatrix}, B = \begin{pmatrix} -5 \\ 6 \end{pmatrix} \quad [A \cdot B = \begin{pmatrix} 14 \\ 35 \\ -9 \end{pmatrix}, \text{ súčin } B \cdot A \text{ neexistuje}]$$

$$\text{d) } A = \begin{pmatrix} 3 & 1 & 2 \\ 1 & 2 & 3 \\ 3 & 6 & 10 \end{pmatrix}, B = \begin{pmatrix} 2 & 3 & -6 \\ 4 & 9 & 7 \\ 6 & 4 & 0 \end{pmatrix} \quad [A \cdot B = \begin{pmatrix} 22 & 26 & -11 \\ 28 & 33 & 8 \\ 90 & 103 & 24 \end{pmatrix}, B \cdot A = \begin{pmatrix} -9 & -28 & -47 \\ 42 & 64 & 105 \\ 22 & 14 & 24 \end{pmatrix}]$$

7. Zistite hodnotu matice:

$$A = \begin{pmatrix} 32 & 6 \\ -4 & 1 \end{pmatrix} \quad B = \begin{pmatrix} -1 & 4 & -6 \\ -3 & 0 & -4 \\ 4 & 6 & 5 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 4 & 6 \\ 1 & -4 & -6 \\ 3 & -8 & -12 \end{pmatrix}$$

$$[h(A) = 2, h(B) = 3, h(C) = 2]$$

8. Zistite hodnotu matice:

$$A = \begin{pmatrix} 2 & 4 \\ -1 & 5 \\ 3 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 1 & 2 & 4 & 3 \\ 1 & 3 & 2 & 4 \\ 1 & 3 & 5 & 7 \\ 1 & 0 & 2 & 1 \\ 1 & 2 & 0 & 1 \\ 1 & 2 & 1 & 2 \end{pmatrix} \quad C = \begin{pmatrix} 2 & 1 & 4 & 1 \\ 1 & 1 & -3 & 2 \\ 3 & 3 & 2 & 1 \\ 4 & -1 & 2 & 4 \end{pmatrix}$$

$$[h(A) = 2, h(B) = 4, h(C) = 4]$$

9. Zistite inverzné matice k daným maticiam:

$$A = \begin{pmatrix} 1 & 3 \\ 5 & 6 \end{pmatrix} \quad B = \begin{pmatrix} 3 & 5 & 8 \\ 1 & 2 & 3 \\ 3 & 6 & 8 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 3 & -2 \\ 3 & 8 & -4 \end{pmatrix}$$

$$[A^{-1} = \begin{pmatrix} -\frac{2}{3} & \frac{1}{3} \\ \frac{5}{9} & -\frac{1}{9} \end{pmatrix}, B^{-1} = \begin{pmatrix} 2 & -8 & 1 \\ -1 & 0 & 1 \\ 0 & 3 & -1 \end{pmatrix}, C^{-1} = \begin{pmatrix} 4 & 0 & -1 \\ 2 & -1 & 0 \\ 7 & -2 & -1 \end{pmatrix}]$$

10. Zistite inverzné matice k daným maticiam:

$$B = \begin{pmatrix} 1 & 6 & 9 \\ 0 & 1 & 2 \\ 0 & 0 & 1 \end{pmatrix} \quad C = \begin{pmatrix} 1 & 3 & -4 \\ 2 & 0 & 1 \\ 0 & 0 & 1 \end{pmatrix} \quad [B^{-1} = \begin{pmatrix} 1 & -6 & 3 \\ 0 & 1 & -2 \\ 0 & 0 & 1 \end{pmatrix}, C^{-1} = \begin{pmatrix} 0 & \frac{1}{2} & -\frac{1}{2} \\ \frac{1}{3} & -\frac{1}{6} & \frac{3}{2} \\ 0 & 0 & 1 \end{pmatrix}]$$